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Some results on g-regular and g-normal spaces

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Abstract. In this paper, map theorem and topological sum theorem on g-regular (resp. g-normal) spaces are given respectively, and their properties are discussed. In addition, Urysohn's Lemma on g-normal spaces is proved.

1. Introduction and preliminaries

It is well-known that g-open subsets in topological spaces are generalized open sets [**3**]. Their complements are said to be g-closed sets which were introduced by Levine in [**8**]. g-regular and g-normal spaces, which are related to g-closed sets, were introduced or investigated in [**11**], [**12**], [**13**], [**14**], etc., In this paper, we give respectively map theorem and topological sum theorem on g-regular (resp. g-normal) spaces, and show that g-regular Lindelöf spaces are g-normal. In addition, we obtain Urysohn's Lemma on g-normal spaces.

In this paper, spaces always mean topological spaces with no separation properties assumed, and maps are onto. 2^X denotes the power set of X. Let (X,τ) be a space. If $A \subset X$, $cl(A)$ and $int(A)$ denotes the closure of A in (X,τ). If $A \subset Y \subset X$, τ_Y denotes $\{U \bigcap Y : U \in \tau\}$, $cl_Y(A)$ and $int_Y(A)$ will respectively denote the closure of A in (Y,τ_Y).

We recall some basic definitions and notations. Let X be a space and let $A \subset X$. A is called g-closed in X [**8**], if $cl(A) \subset U$ whenever U is open and $A \subset U$; A is called g-open in X [**8**], if $X - A$ is g-closed in X. X is called a g-regular space [**12**], if for each pair consisting of a point x and a g-closed subset F not containing x, there exist disjoint open subsets U and V such that $x \in U$ and $F \subset V$. X is called a g-normal space [**11**], if for each pair consisting of disjoint g-closed subsets A and B, there exist disjoint open subsets U and V such that $A \in U$ and $B \subset V$.

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Let $f : X \to Y$ be a map. f is called a perfect map, if f is a continuous and closed map, and $f^{-1}(y)$ is compact for any $y \in Y$. f is called a g-continuous map [**2**], if $f^{-1}(V)$ is g-open in X for each open subset V of Y.

2. Related results of g-regular spaces

LEMMA 2.1 ([**10**]). *If $f : X \to Y$ is a map, $A \subset X$ and $B \subset Y$. then $f^{-1}(B) \subset A$ if and only if $B \subset Y - f(X - A)$.*

LEMMA 2.2. *Let $f : (X, \tau) \longrightarrow (Y, \sigma)$ be a g-continuous and closed map. If B is a g-closed subset of Y, then $f^{-1}(B)$ is a g-closed subset of X.*

PROOF. Suppose $f^{-1}(B) \subset U \in \tau$, then $B \subset Y - f(X - U)$ by Lemma 2.1. Since f is a closed map, then $Y - f(X - U) \in \sigma$. B is g-closed in Y implies that $cl(B) \subset Y - f(X - U)$. By Lemma 2.1, $f^{-1}(cl(B)) \subset U$. Since f is g-continuous, then $f^{-1}(cl(B))$ is g-closed in X. Thus $cl(f^{-1}(cl(B))) \subset U$. Hence $cl(f^{-1}(B) \subset U$. Therefore, $f^{-1}(B)$ is g-closed in X. □

THEOREM 2.1. *Let $f : X \to Y$ be a g-continuous and closed map, and $f^{-1}(y)$ is compact for any $y \in Y$. If X is g-regular, then Y is also g-regular.*

PROOF. Suppose $y \notin B$ and B is g-closed in Y, then $f^{-1}(B))$ is g-closed in X by Lemma 2.2. $y \notin B$ implies that $f^{-1}(y) \cap f^{-1}(B) = \emptyset$. For every $x \in f^{-1}(y)$, $x \notin f^{-1}(B)$, since X is g-regular, then there exist disjoint open subsets U_x and V_x of X such that $x \in U_x$ and $f^{-1}(B) \subset V_x$. Since $\{U_x : x \in f^{-1}(y)\}$ is a open cover of set $f^{-1}(y)$ and $f^{-1}(y)$ is compact, then $\{U_x : x \in f^{-1}(y)\}$ has a finite subcover $\{U_{x_i} : i \leqslant n\}$. Put

$$U = \bigcup_{i=1}^{n} U_{x_i}, \quad V = \bigcap_{i=1}^{n} V_{x_i}.$$

Then U, V are disjoint open subsets of X, $f^{-1}(y) \subset U$ and $f^{-1}(B) \subset V$. By Lemma 2.1, $y \in Y - f(X - U)$ and $B \subset Y - f(X - V)$. Let $G = Y - f(X - U)$ and $W = Y - f(X - V)$, then G, W are open in Y. $U \cap V = \emptyset$ implies that $(X - U) \cup (X - V) = X - U \cap V = X$. Thus $W \cap G = \emptyset$. Therefore, $(Y, \sigma, \mathcal{I})$ is g-regular. □

COROLLARY 2.1. *Let $f : X \to Y$ be a perfect map. If X is g-regular, then Y is also g-regular.*

THEOREM 2.2. *g-regular Lindelöf spaces are g-normal spaces.*

PROOF. Suppose X is a g-regular Lindelöf space. For each pair of disjoint g-closed subsets A and B of X, $x \in A$ implies $x \notin B$. Since X is g-regular, then there exists disjoint open subsets U_x and W_x of X such that $x \in U_x$ and $B \subset W_x$. Now $U_x \bigcap W_x = \emptyset$ implies $cl(U_x) \bigcap W_x = \emptyset$. So $cl(U_x) \bigcap B = \emptyset$. $\mathcal{U} = \{U_x : x \in A\}$ is an open cover of set A. $A \subset \bigcup_{x \in A} U_x$, since A is g-closed in X, then $cl(A) \subset \bigcup_{x \in A} U_x$. So $\mathcal{U} \cup \{X - cl(A)\}$ is an open cover of X. Note that X is a Lindelöf space. Thus $\mathcal{U} \cup \{X - cl(A)\}$ has a countable subcover $\{U_n : n \in N\} \cup \{X - cl(A)\}$. So $X = (\bigcup_{n=1}^{\infty} U_n) \bigcup (X - cl(A))$. Hence $A \subset cl(A) \subset \bigcup_{n=1}^{\infty} U_n$, where $cl(U_n) \bigcap B = \emptyset$ for any $n \in N$.

$y \in B$ implies $y \notin A$. Since X is g-regular, then there exists disjoint open subsets V_y and L_y of X such that $y \in V_y$ and $A \subset L_y$. Now $V_y \bigcap L_y = \emptyset$ implies $cl(V_y) \bigcap L_y = \emptyset$. So $cl(V_y) \bigcap A = \emptyset$. $\mathcal{V} = \{V_y : y \in B\}$ is an open cover of set B.

Similarly, there exists a countable subset $\{V_n : n \in N\}$ of $\mathcal{V}$ such that $B \subset \bigcup\limits_{n=1}^{\infty} V_n$, where $cl(V_n) \bigcap A = \emptyset$ for any $n \in N$.

Put

$$G_n = U_n - \bigcup_{i=1}^{n} cl(V_i), \quad G = \bigcup_{n=1}^{\infty} G_n,$$
$$W_n = V_n - \bigcup_{i=1}^{n} cl(U_i), \quad W = \bigcup_{n=1}^{\infty} W_n.$$

Obviously, for each $n \in N$, G_n and W_m are open in X. So G and W are open in X.

Claim : for any $n, m \in N$, $G_n \cap W_m = \emptyset$.

(1) If $m \leqslant n$, then $W_m \subset V_m \subset \bigcup\limits_{i=1}^{m} cl(V_i) \subset \bigcup\limits_{i=1}^{n} cl(V_i)$. Since $G_n \bigcap \bigcup\limits_{i=1}^{n} cl(V_i) = \emptyset$, then $G_n \bigcap W_m = \emptyset$. (2) If $m > n$, then $G_n \subset U_n \subset \bigcup\limits_{i=1}^{n} cl(U_i) \subset \bigcup\limits_{i=1}^{m} cl(U_i)$. Since $W_m \bigcap \bigcup\limits_{i=1}^{m} cl(U_i) = \emptyset$, then $W_m \bigcap G_n = \emptyset$. Thus, for any $n, m \in N$, $G_n \cap W_m = \emptyset$.

Therefore, $G \bigcap W = \bigcap\limits_{n,\ m=1}^{\infty} (G_n \cap W_m) = \emptyset$.

We will prove that $A \subset G$ and $B \subset W$.

For $x \in A$, $A \subset \bigcup\limits_{i=1}^{\infty} U_n$ implies that $x \in U_n$ for some $n \in N$.

Since $cl(V_i) \cap A = \emptyset$ for any $i \in N$, then $x \notin cl(V_i)$ for any $i \in N$. So $x \notin \bigcup\limits_{i=1}^{n} cl(V_i)$. Thus $x \in G_n$, so $x \in G$. Therefore $A \subset G$.

The proof of $B \subset W$ is similar. □

Lemma 2.3. *Let (X, τ) be a space. Then*

(1) If $A \subset Y \subset X$, A is g-closed in Y and Y is closed in X, then A is g-closed in X.

(2) If $A \subset Y \subset X$, A is g-closed in X, then A is g-closed in Y.

(3) If B, $Y \subset X$, B is g-closed in X and Y is closed in X, then $B \cap Y$ is g-closed in X.

Proof. (1) Suppose $A \subset U \in \tau$, then $A \subset U \cap Y \in \tau_Y$. Since A is g-closed in Y, then $cl(A) \cap Y = cl_Y(A) \subset U \cap Y$. Since $A \subset Y$ and Y is closed in X, then $cl(A) \subset Y$. Thus $cl(A) \subset U \cap Y \subset U$. Therefore A is g-closed in X.

(2) Suppose $A \subset U \in \tau_Y$, then $U = V \cap Y$ for some $V \in \tau$. Now $A \subset V \in \tau$. Since A is g-closed in X, then $cl(A) \subset V$. Thus $cl_Y(A) = cl(A) \cap Y \subset V \cap Y = U$. Therefore A is g-closed in Y.

(3) Suppose $B \cap Y \subset U \in \tau$, then $B \subset U \cup (X - Y) \in \tau$. Since B is g-closed in X, then $cl(B) \subset U \cup (X - Y)$. Thus $cl(B \cap Y) \subset cl(B) \cap cl(Y) = cl(B) \cap Y \subset (U \cup (X - Y)) \cap Y = U \cap Y \subset U$. Therefore $B \cap Y$ is g-closed in Y. □

LEMMA 2.4 ([**12**]). *If (X, τ) is g-regular and Y is closed in X, then Y is g-regular.*

THEOREM 2.3. *Let $\{X_\alpha : \alpha \in \Lambda\}$ be a family of pairwise disjoint spaces. Then $\bigoplus_{\alpha\in\Lambda} X_\alpha$ is a g-regular space if and only if every X_α is a g-regular space.*

PROOF. The proof of Necessity follows from Lemma 2.7.

Sufficiency. Let $X = \bigoplus_{\alpha\in\Lambda} X_\alpha$ and let $x \notin F$ and F be g-closed in X. Since every X_α is open-and-closed in X, then for any $\alpha \in \Lambda$, $F \cap X_\alpha$ is g-closed in X_α by Lemma 2.6. Obviously, there exists $\beta \in \Lambda$ such that $x \in X_\beta$. Since X_β is g-regular, then there exist disjoint open subsets U and V of X_β such that $x \in U$ and $F \cap X_\beta \subset V$. So $F \subset V \cup (X - X_\beta)$. Since X_β is open-and-closed in X, then U and $V \cup (X - X_\beta)$ are disjoint open subsets of X. Therefore X is g-regular. □

3. Related results of g-normal spaces

THEOREM 3.1. *X is g-normal if and only if for each g-closed subset F of X and g-open subset W of X containing F, there exists a sequence $\{U_n\}$ of open subsets of X such that $F \subset \bigcup_{n=1}^{\infty} U_n$ and $cl(U_n) \subset W$ for any $n \in N$.*

PROOF. The proof of Necessity is obvious.

Sufficiency. Suppose A and B are disjoint g-closed subsets of X. Let $F = A$ and $W = X - B$, by hypothesis, there exists sequence $\{U_n\}$ of open subsets of X such that

$$A \subset \bigcup_{n=1}^{\infty} U_n \quad \text{and} \quad cl(U_n) \cap B = \emptyset \text{ for any } n \in N.$$

And let $F = B$ and $W = X - A$, by hypothesis, there exists a sequence $\{V_n\}$ of open subsets of X such that

$$B \subset \bigcup_{n=1}^{\infty} V_n \quad \text{and} \quad cl(V_n) \cap A = \emptyset \text{ for any } n \in N.$$

Put

$$G_n = U_n - \bigcup_{i=1}^{n} cl(V_i), \quad G = \bigcup_{n=1}^{\infty} G_n,$$
$$H_n = V_n - \bigcup_{i=1}^{n} cl(U_i), \quad H = \bigcup_{n=1}^{\infty} H_n.$$

Obviously, for each $n \in N$, G_n and H_m are open in X. So G and H are open in X.

By a similar way as in the proof of Theorem 2.5, we can prove that $G \bigcap H = \emptyset$, $A \subset G$ and $B \subset H$. □

Below we give Urysohn's Lemma on g-normal spaces.

THEOREM 3.2. *X is g-normal spaces if and only if for each pair of disjoint g-closed subsets A and B of X, there exists a continuous mapping $f : X \to [0, 1]$ such that $f(A) = \{0\}$ and $f(B) = \{1\}$.*

PROOF. Sufficiency. Suppose for each pair of disjoint g-closed subsets A and B of X, there exists a continuous mapping $f : X \to [0, 1]$ such that $f(A) = \{0\}$ and

$f(B) = \{1\}$. Put $U = f^{-1}([0, 1/2))$, $V = f^{-1}((1/2, 1])$, then U and V are disjoint open subsets of X such that $A \subset U$ and $B \subset V$. Hence X is g-normal.

Necessity. Suppose X is g-normal. For each pair of disjoint g-closed subsets A and B of X, $A \subset X - B$, where A is g-closed and $X - B$ in X is g-open in X, by Corollary 2.12 in [**14**], there exists an open subset $U_{1/2}$ of X such that

$$A \subset U_{1/2} \subset cl(U_{1/2}) \subset X - B.$$

Since $A \subset U_{1/2}$, A is g-closed in X and $U_{1/2}$ is g-open in X, then there exists an open subset $U_{1/4}$ of X such that $A \subset U_{1/4} \subset cl(U_{1/4}) \subset U_{1/2}$ by Corollary 2.12 in [**14**]. Since $cl(U_{1/2}) \subset X - B$, $cl(U_{1/2})$ is g-closed in X and $X - B$ is g-open in X, then there exists an open subset $U_{3/4}$ of X such that $cl(U_{1/2}) \subset U_{3/4} \subset cl(U_{3/4}) \subset X - B$ by Corollary 2.12 in [**14**]. Thus, there exist two open subsets $U_{1/2}$ and $U_{3/4}$ of X such that

$$A \subset U_{1/4} \subset cl(U_{1/4}) \subset U_{1/2} \subset cl(U_{1/2}) \subset U_{3/4} \subset cl(U_{3/4}) \subset X - B.$$

We get a family $\{U_{m/2^n} : 1 \leqslant m < 2^n, n \in N\}$ of open subsets of X, denotes $\{U_{m/2^n} : 1 \leqslant m < 2^n, n \in N\}$ by $\{U_\alpha : \alpha \in \Gamma\}$. $\{U_\alpha : \alpha \in \Gamma\}$ satisfies the following condition:

(1) $A \subset U_\alpha \subset cl(U_\alpha) \subset X - B$,

(2) if $\alpha < \alpha'$, then $cl(U_\alpha) \subset U_{\alpha'}$.

We define $f : X \to [0, 1]$ as follows:

$$f(x) = \begin{cases} inf\{\alpha \in \Gamma : x \in U_\alpha\}, & \text{if } x \in U_\alpha \text{ for some } \alpha \in \Gamma, \\ 1, & \text{if } x \notin U_\alpha \text{ for any } \alpha \in \Gamma. \end{cases}$$

For each $x \in A$, $x \in U_\alpha$ for any $\alpha \in \Gamma$ by (1), so $f(x) = inf\{\alpha \in \Gamma : x \in U_\alpha\} = inf\Gamma = 0$. Thus, $f(A) = \{0\}$.

For each $x \in B$, $x \notin X - B$ implies $x \notin U_\alpha$ for any $\alpha \in \Gamma$ by (1), so $f(x) = 1$. Thus, $f(B) = \{1\}$.

We have to show f is continuous.

For $x \in X$ and $\alpha \in \Gamma$, we have the following Claim:

Claim 1: if $f(x) < \alpha$, then $x \in U_\alpha$.

Suppose $f(x) < \alpha$, then $inf\{\alpha \in \Gamma : x \in U_\alpha\} < \alpha$, so there exists $\alpha_1 \in \{\alpha \in \Gamma : x \in U_\alpha\}$ such that $\alpha_1 < \alpha$. By (2), $cl(U_{\alpha_1}) \subset U_\alpha$. Notice that $x \in U_{\alpha_1}$. Hence $x \in U_\alpha$.

Claim 2: if $f(x) > \alpha$, then $x \notin cl(U_\alpha)$.

Suppose $f(x) > \alpha$, then there exists $\alpha_1 \in \Gamma$ such that $\alpha < \alpha_1 < f(x)$. Notice that $\alpha_1 \in \{\alpha \in \Gamma : x \in U_\alpha\}$ implies $\alpha_1 \geqslant inf\{\alpha \in \Gamma : x \in U_\alpha\} = f(x)$. Thus, $\alpha_1 \notin \{\alpha \in \Gamma : x \in U_\alpha\}$. So $x \notin U_{\alpha_1}$. By (2), $cl(U_\alpha) \subset U_{\alpha_1}$. Hence $x \notin cl(U_\alpha)$.

Claim 3: if $x \notin cl(U_\alpha)$, then $f(x) \geqslant \alpha$.

Suppose $x \notin cl(U_\alpha)$, we claim that $\alpha < \beta$ for any $\beta \in \{\alpha \in \Gamma : x \in U_\alpha\}$. Otherwise, there exists $\beta \in \{\alpha \in \Gamma : x \in U_\alpha\}$ such that $\alpha \geqslant \beta$. $x \notin cl(U_\alpha)$ implies $\alpha \notin \{\alpha \in \Gamma : x \in U_\alpha\}$. So $\alpha \neq \beta$. Thus $\alpha > \beta$. By (2), $cl(U_\beta) \subset U_\alpha$. So $x \notin \beta$, contridiction. Therefore $\alpha < \beta$ for any $\beta \in \{\alpha \in \Gamma : x \in U_\alpha\}$. Hence $\alpha \leqslant inf\{\alpha \in \Gamma : x \in U_\alpha\} = f(x)$.

For $x_0 \in X$, if $f(x_0) \in (0,1)$, suppose V is an open neighborhood of $f(x_0)$ in $[0,1]$, then there exists $\varepsilon > 0$ such that $(f(x_0)-\epsilon, f(x_0)+\epsilon) \subset V \bigcap (0,1)$. Pick $\alpha', \alpha'' \in \Gamma$ such that

$$0 < f(x_0) - \epsilon < \alpha' < f(x_0) < \alpha'' < f(x_0) + \epsilon < 1.$$

By **Claim 1** and **Claim 2**, $x_0 \in U''_\alpha$, $x_0 \notin cl(U'_\alpha)$. Put $U = U''_\alpha - cl(U'_\alpha)$, then U is an open neighborhood of x_0 in X.

We will prove that $f(U) \subset (f(x_0)-\epsilon, f(x_0)+\epsilon)$. if $y \in f(U)$, then $y = f(x)$ for some $x \in U$. $x \in U$ implies that $x \in U''_\alpha$ and $x \notin cl(U'_\alpha)$. Since $x \in U''_\alpha$, then $\alpha'' \in \{\alpha \in \Gamma : x \in U_\alpha\}$. Thus, $\alpha'' \geqslant inf\{\alpha \in \Gamma : x \in U_\alpha\} = f(x)$. Notice that $\alpha'' < f(x_0)+\epsilon$. Therefore $f(x) < f(x_0)+\epsilon$. Since $x \notin cl(U'_\alpha)$, then $f(x) \geqslant \alpha'$ by **Claim 3**. Notice that $f(x_0) - \epsilon < \alpha'$. Therefore $f(x) > f(x_0) - \epsilon$. Hence, $f(U) \subset (f(x_0)-\epsilon, f(x_0)+\epsilon)$.

Therefore, $f(U) \subset V$. This implies f is continuous at x_0.

if $f(x_0) = 0$, or 1, the proof that f is continuous at x_0 is similar. □

Theorem 3.3. *Let $f : X \to Y$ be a g-continuous and closed map. If X is g-normal, then Y is g-normal.*

Proof. Suppose A and B are disjoint g-closed subsets of Y, then $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint g-closed subsets of X by Lemma 2.2. Since X is g-normal, then exist disjoint open subsets U and V of X such that $f^{-1}(A) \subset U$ and $f^{-1}(B) \subset V$. By Lemma 2.1, $A \subset Y - f(X-U)$ and $B \subset Y - f(X-V)$. Note that $Y - f(X-U)$ and $Y - f(X-V)$ are disjoint open subsets of Y. Hence X is g-normal. □

Corollary 3.1. *Let $f : X \to Y$ be a continuous and closed map. If X is g-normal, then Y is g-normal.*

Theorem 3.4. *Let $\{X_\alpha : \alpha \in \Lambda\}$ be a family of pairwise disjoint spaces. Then $\bigoplus_{\alpha \in \bigwedge} X_\alpha$ is a g-normal space if and only if every X_α is a g-normal space.*

Proof. The proof of Necessity follows from that fact that the g-normality is closed heredity.

Sufficiency. Let $X = \bigoplus_{\alpha\in\bigwedge} X_\alpha$ and let A and B be disjoint g-closed subsets of X. Then for any $\alpha \in \Lambda$, $A \cap X_\alpha$ and $B \cap X_\alpha$ are disjoint g-closed subsets of X_α by Lemma 2.8. Since X_α is g-regular, then there exist disjoint open subsets U_α and V_α of X_α such that $A \cap X_\alpha \subset U_\alpha$ and $B \cap X_\alpha \subset V_\alpha$.

Clearly,

$$A = A \bigcap X = A \bigcap (\bigcup_{\alpha\in\bigwedge} X_\alpha) \subset U = \bigcup_{\alpha\in\bigwedge} U_\alpha,$$
$$B = B \bigcap X = B \bigcap (\bigcup_{\alpha\in\bigwedge} X_\alpha) \subset V = \bigcup_{\alpha\in\bigwedge} V_\alpha.$$

If $\alpha \neq \beta$, then $U_\alpha \cap V_\beta \subset X_\alpha \cap X_\beta = \emptyset$. Thus for any $\alpha, \beta \in \Lambda$, $U_\alpha \cap V_\beta = \emptyset$. Hence $U \bigcap V = \bigcap_{\alpha,\ \beta\in\Lambda} (U_\alpha \cap V_\beta) = \emptyset$.

Since every X_α is open in X, then U and V are open in X. Therefore X is g-normal. □

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