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On pseudo cyclic Ricci symmetric manifolds admitting semi-symmetric metric connection

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Abstract. The object of the present paper is to investigate the applications of pseudo cyclic Ricci symmetric manifolds admitting a semi-symmetric metric connection to the general relativity and cosmology.

1. Introduction

A Riemannian manifold is Ricci symmetric if its Ricci tensor S of type (0, 2) satisfies $\nabla S = 0$, where ∇ denotes the Riemannian connection. During the last five decades, the notion of Ricci symmetry have been weakened by many authors such as Ricci-recurrent manifolds [**11**], Ricci semi-symmetric manifolds [**14**], pseudo Ricci symmetric manifolds by M. C. Chaki [**4**]. A non-flat Riemannian manifold (M^n, g) is said to be pseudo Ricci symmetric [**4**] if its Ricci tensor S of type (0, 2) is not identically zero and satisfies the condition

$$(\nabla_X S)(Y, Z) = 2A(X)S(Y, Z) + A(Y)S(X, Z) + A(Z)S(Y, X), \tag{1.1}$$

where A is a nowhere vanishing 1-form and ∇ denotes the operator of covariant differentiation with respect to the metric tensor g. Such an n-dimensional manifold is denoted by $(PRS)_n$.

Extending the notion of pseudo Ricci symmetric manifold, recently the present authors [**13**] introduced the notion of *pseudo cyclic Ricci symmetric manifold*. A Riemannian manifold $(M^n, g)(n > 2)$ is called *pseudo cyclic Ricci symmetric manifold* if its Ricci tensor S of type (0, 2) is not identically zero and satisfies the following:

$$\begin{aligned}(\nabla_X S)(Y, Z) + (\nabla_Y S)(Z, X) + (\nabla_Z S)(X, Y)\\ = 2A(X)S(Y, Z) + A(Y)S(X, Z) + A(Z)S(Y, X),\end{aligned} \tag{1.2}$$

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where A is a nowhere vanishing 1-form associated to the vector field ρ such that $A(X) = g(X, \rho)$ for all X. Such an n-dimensional manifold is denoted by $(PCRS)_n$. Every $(PRS)_n$ is $(PCRS)_n$ but not conversely [**13**].

Section 2 is concerned with preliminaries of semi-symmetric metric connection along with its physical significance. In general relativity the matter content of the spacetime is described by the energy-momentum tensor T which is to be determined from the physical considerations dealing with the distribution of matter and energy. Since the matter content of the universe is assumed to behave like a perfect fluid in the standard cosmological models, the physical motivation for studying Lorentzian manifolds is the assumption that a gravitational field may be effectively modeled by some Lorentzian metric defined on a suitable four dimensional manifold M. The Einstein equations are fundamental in the construction of cosmological models which imply that the matter determines the geometry of the spacetime and conversely the motion of matter is determined by the metric tensor of the space which is non-flat.

The present paper deals with $(PCRS)_n$ admitting a semi-symmetric metric connection. A $(PCRS)_n$ admitting semi-symmetric metric connection is denoted by $[(PCRS)_n, \tilde{\nabla}]$. In section 3 we investigate the applications of $[(PCRS)_n, \tilde{\nabla}]$ to the general relativity and cosmology. It is shown that a viscous fluid spacetime obeying Einstein's equation with a cosmological constant is a connected Lorentzian $[(PCRS)_4, \tilde{\nabla}]$. Consequently $[(PCRS)_4, \tilde{\nabla}]$ can be viewed as a model of the viscous fluid spacetime. Also it is observed that in a viscous fluid $[(PCRS)_4, \tilde{\nabla}]$ spacetime none of the isotropic pressure and energy density can be a constant and the matter content of the spacetime is a non-thermalised fluid under a certain condition.

The physical motivation for studying various types of spacetime models in cosmology is to obtain the information of different phases in the evolution of the universe, which may be classified into three phases, namely, the initial phase, the intermediate phase and the final phase. The initial phase is just after the Big Bang when the effects of both viscosity and heat flux were quite pronounced. The intermediate phase is that when the effect of viscosity was no longer significant but the heat flux was till not negligible. The final phase, which extends to the present state of the universe when both the effects of viscosity and heat flux have become negligible and the matter content of the universe may be assumed to be perfect fluid. The study of $[(PCRS)_4, \tilde{\nabla}]$ is important because such spacetime represents the third phase in the evolution of the universe. Consequently the investigations of $[(PCRS)_n, \tilde{\nabla}]$ helps us to have a deeper understanding of the global character of the universe including the topology, because the nature of the singularities can be defined from a differential geometric stand point.

2. Semi-symmetric metric connection

In 1924 Friedmann and Schouten [**6**] introduced the notion of semi-symmetric linear connection on a differentiable manifold. Then in 1932 Hayden [**7**] introduced the idea of metric connection with torsion on a Riemannian manifold. A systematic study of the semi-symmetric metric connection on a Riemannian manifold has been given by K. Yano in 1970 [**15**]. Semi-symmetric metric connection on Riemannian manifolds is also studied by various authors such as ([**1**],[**5**],[**8**]).

Semi-symmetric metric connection plays an important role in the study of Riemannian manifolds. There are various physical problems involving the semi-symmetric metric connection. For example, if a man is moving on the surface of the earth always facing one definite point, say Jerusalem or Mekka or the North Pole, then this displacement is semi-symmetric and metric ([**12**], p.143). Again during the mathematical congress in Moscow in 1934 one evening mathematicians invented the "Moscow displacement". The streets of Moscow are approximately straight lines through the Kremlin and concentric circles around it. If a person walk in the street always facing the Kremlin, then this displacement is semi-symmetric and metric ([**12**], p.143).

Let (M^n, g) be an n-dimensional Riemannian manifold of class C^∞ with the metric tensor g and ∇ be the Riemannian connection of the manifold (M^n, g). A linear connection $\tilde{\nabla}$ on (M^n, g) is said to be semi-symmetric [**6**] if the torsion tensor τ of the connection $\tilde{\nabla}$ satisfies

$$\tau(X,Y) = \alpha(Y)X - \alpha(X)Y \tag{2.1}$$

for any vector field X, Y on M and α is an 1-form associated with the torsion tensor τ of the connection $\tilde{\nabla}$ given by

$$\alpha(X) = g(X, \rho),$$

ρ being the vector field associated with the 1-form α. The 1-form α is called the associated 1-form of the semi-symmetric connection and the vector field ρ is called the associated vector field of the connection. A semi-symmetric connection $\tilde{\nabla}$ is called a semi-symmetric metric connection [**7**] if in addition it satisfies

$$\tilde{\nabla} g = 0. \tag{2.2}$$

The relation between the semi-symmetric connection $\tilde{\nabla}$ and the Riemannian connection ∇ of (M^n, g) is given by [**15**]

$$\tilde{\nabla}_X Y = \nabla_X Y + \alpha(Y)X - g(X,Y)\rho. \tag{2.3}$$

In particular, if the 1-form α vanishes identically then a semi-symmetric metric connection reduces to the Riemannian connection. The covariant differentiation of an 1-form ω with respect to $\tilde{\nabla}$ is given by [**15**]

$$(\tilde{\nabla}_X \omega)(Y) = (\nabla_X \omega)(Y) + \omega(X)\alpha(Y) - \omega(\rho)g(X,Y).$$

If R and $\tilde{R}$ are respectively the curvature tensor of the Levi-Civita connection ∇ and the semi-symmetric metric connection $\tilde{\nabla}$, then we have [**15**]

$$\tilde{R}(X,Y)Z = R(X,Y)Z - P(Y,Z)X + P(X,Z)Y - g(Y,Z)LX + g(X,Z)LY, \tag{2.4}$$

where P is a tensor field of type (0,2) given by

$$P(X,Y) = g(LX,Y) = (\nabla_X \alpha)(Y) - \alpha(X)\alpha(Y) + \frac{1}{2}\alpha(\rho)g(X,Y) \tag{2.5}$$

for any vector fields X and Y. From (2.4), it follows that

$$\tilde{S}(Y,Z) = S(Y,Z) - (n-2)P(Y,Z) - ag(Y,Z), \tag{2.6}$$

where $\tilde{S}$ and S denote respectively the Ricci tensor with respect to $\tilde{\nabla}$ and ∇, $a =$ trace P. The tensor P of type (0,2) given in (2.5) is not symmetric in general and hence from (2.6) it follows that the Ricci tensor $\tilde{S}$ of the semi-symmetric metric connection $\tilde{\nabla}$ is not so. But if we consider that the 1-form α, associated with the torsion tensor τ, is closed then it can be easily shown that the relation

$$(\nabla_X \alpha)(Y) = (\nabla_Y \alpha)(X) \quad \text{for all vector fields } X, Y$$

holds and hence the tensor $P(X,Y)$ is symmetric. Consequently, the Ricci tensor $\tilde{S}$ is symmetric. Conversely, if $P(X,Y)$ is symmetric then from (2.5) it follows that the 1-form α is closed. This leads to the following:

PROPOSITION 2.1. [**2**] *Let $(M^n, g)(n > 2)$ be a Riemannian manifold admitting a semi-symmetric metric connection $\tilde{\nabla}$. Then the Ricci tensor $\tilde{S}$ of $\tilde{\nabla}$ is symmetric if and only if the 1-form α, associated with the torsion tensor τ is closed.*

Contracting (2.6) with respect to Y and Z, it can be easily found that

$$\tilde{r} = r - 2(n-1)a, \tag{2.7}$$

where $\tilde{r}$ and r denote respectively the scalar curvature with respect to $\tilde{\nabla}$ and ∇.

DEFINITION 2.1. A Riemannian manifold $(M^n, g)(n > 2)$ is called pseudo cyclic Ricci symmetric manifold admitting semi-symmetric metric connection if its Ricci tensor $\tilde{S}$ of type (0,2) is not identically zero and satisfies the condition

$$\begin{aligned}(\tilde{\nabla}_X \tilde{S})(Y,Z) + (\tilde{\nabla}_Y \tilde{S})(Z,X) + (\tilde{\nabla}_Z \tilde{S})(X,Y) \\ = 2\tilde{A}(X)\tilde{S}(Y,Z) + \tilde{A}(Y)\tilde{S}(Z,X) + \tilde{A}(Z)\tilde{S}(X,Y),\end{aligned} \tag{2.8}$$

where $\tilde{A}$ is a nowhere vanishing 1-form and $\tilde{\nabla}$ denotes the semi-symmetric metric connection. Such an n-dimensional manifold is denoted by $[(PCRS)_n, \tilde{\nabla}]$.

In view of (2.3) and (2.6), it follows from (2.8) that

$$\begin{aligned}&(\nabla_X S)(Y,Z) + (\nabla_Y S)(Z,X) + (\nabla_Z S)(X,Y) \\ &= 2[\tilde{A}(X) - \alpha(X)]S(Y,Z) + [\tilde{A}(Y) - \alpha(Y)]S(Z,X) + [\tilde{A}(Z) - \alpha(Z)]S(X,Y) \\ &+(n-2)[(\nabla_X P)(Y,Z) + (\nabla_Y S)(Z,X) + (\nabla_Z S)(X,Y) + \{\alpha(X) \\ &-2\tilde{A}(X)\}P(Y,Z) + \{\alpha(Y) - \tilde{A}(Y)\}P(Z,X) + \{\alpha(Z) - \tilde{A}(Z)\}P(X,Y) \\ &-g(X,Y)P(Z,\rho) - g(Y,Z)P(X,\rho) - g(Z,X)P(Y,\rho)] + [da(X) + \alpha(QX) \\ &-2a\tilde{A}(X)]g(Y,Z) + [da(Y) + \alpha(QY) - a\tilde{A}(Y)]g(Z,X) \\ &+[da(Z) + \alpha(QZ) - a\tilde{A}(Z)]g(X,Y),\end{aligned} \tag{2.9}$$

where Q is the Ricci-operator, i.e., $S(X,Y) = g(QX,Y)$.

If, in particular, the 1-form α vanishes identically, then (2.9) reduces to (1.2) with $\tilde{A} = A$. Hence the manifold $(PCRS)_n$ is a particular case of $[(PCRS)_n, \tilde{\nabla}]$. Also the manifold $(PCRS)_n$ could be a $[(PCRS)_n, \tilde{\nabla}]$ when it admits a semi-symmetric metric connection $\tilde{\nabla}$ different from the Riemannian connection ∇.

We now prove the following Lemma.

LEMMA 2.1. *If in a $[(PCRS)_n, \tilde{\nabla}]$, the 1-form α, associated with the torsion tensor τ, is closed, then its Ricci tensor $\tilde{S}$ is of the form:*

$$\tilde{S} = \tilde{r}\gamma \otimes \gamma, \tag{2.10}$$

where γ is a nowhere vanishing 1-form defined by $\gamma(X) = g(X, \mu)$, μ being a unit vector field.

Proof. Interchanging X and Y in (2.8), we obtain

$$(\tilde{\nabla}_Y \tilde{S})(X,Z) + (\tilde{\nabla}_X \tilde{S})(Z,Y) + (\tilde{\nabla}_Z \tilde{S})(Y,X) \\ = 2\tilde{A}(Y)\tilde{S}(X,Z) + \tilde{A}(X)\tilde{S}(Z,Y) + \tilde{A}(Z)\tilde{S}(Y,X).$$

Since the 1-form α, associated with the torsion tensor τ, is closed, from Proposition 2.1, it follows that the Ricci tensor $\tilde{S}$ is symmetric. Subtracting the last relation from (2.8), we get

$$\tilde{A}(X)\tilde{S}(Y,Z) = \tilde{A}(Y)\tilde{S}(X,Z). \tag{2.11}$$

Contracting (2.11) with respect to Y and Z, we have

$$\tilde{r}\tilde{A}(X) = \tilde{A}(\tilde{Q}X), \tag{2.12}$$

where $\tilde{Q}$ is the Ricci operator associated with the Ricci tensor $\tilde{S}$, i.e., $\tilde{S}(X,Y) = g(\tilde{Q}X, Y)$.
Setting $Y = \rho$ in (2.11), we get

$$\tilde{S}(X,Z) = \frac{1}{\tilde{A}(\rho)}\tilde{A}(X)\tilde{A}(\tilde{Q}Z), \tag{2.13}$$

which, in view of (2.12), yields

$$\tilde{S}(X,Z) = \tilde{r}\gamma(X)\gamma(Z), \tag{2.14}$$

where $\gamma(X) = g(X,\mu) = \frac{1}{\sqrt{\tilde{A}(\rho)}}\tilde{A}(X)$, μ being a unit vector field associated with the 1-form γ. Hence the Lemma is proved.

Using (2.10) and (2.7) in (2.6), we obtain

$$S(Y,Z) = ag(Y,Z) + [r - 2(n-1)a]\gamma(Y)\gamma(Z) + (n-2)P(Y,Z), \tag{2.15}$$

provided that the 1-form α, associated with the torsion tensor τ, is closed. The unit vector field μ associated to the 1-form γ is called the generator of $[(PCRS)_n, \tilde{\nabla}]$ and P is known as the structure tensor of $[(PCRS)_n, \tilde{\nabla}]$.

A non-zero vector V on a manifold M is said to be timelike (resp. non-spacelike, null, spacelike) if it satisfies $g(V,V) < 0$ (resp. $\geqslant 0, = 0, > 0$) ([**3**],[**10**]).

Since μ is a unit vector field on the connected and non-compact Riemannian manifold $M =[(PCRS)_n, \tilde{\nabla}]$ with metric tensor g, it can be easily shown ([**10**], p.148) that $\tilde{g} = g - 2\gamma \otimes \gamma$ is a Lorentz metric on M. Also μ becomes timelike so the resulting Lorentz manifold is time-orientable.

3. General relativistic viscous fluid $[(PCRS)_4, \tilde{\nabla}]$ spacetime

A viscous fluid spacetime is a connected Lorentzian manifold (M^4, g) with signature (-,+,+,+). In general relativity the key role is played by Einstein's equation

$$S + \left(\lambda - \frac{r}{2}\right) g = kT, \tag{3.1}$$

where S is the Ricci tensor of type (0, 2), r is the scalar curvature, λ is the cosmological constant, k is the gravitational constant and T is the energy-momentum tensor of type (0, 2). Let us consider the energy-momentum tensor T of a viscous fluid spacetime to the following form [**9**]:

$$T = pg + (\sigma + p)\gamma \otimes \gamma + P, \tag{3.2}$$

where σ, p are respectively the energy density, isotropic pressure and P denotes the anisotropic pressure tensor of the fluid, μ is the unit timelike vector field, called flow vector field of the fluid associated with the 1-form γ given by $g(X, \mu) = \gamma(X)$ for all X. Then by virtue of (3.2), (3.1) can be written as

$$S = \left(\frac{r}{2} + kp - \lambda\right) g + k(\sigma + p)\gamma \otimes \gamma + P, \tag{3.3}$$

which, in view of (2.15) shows that the spacetime under consideration is a $[(PCRS)_4, \tilde{\nabla}]$ with μ as the unit timelike flow vector field of the fluid and P as the anisotropic pressure tensor, where the 1-form α, associated with the torsion tensor τ, corresponding to the semi-symmetric metric connection $\tilde{\nabla}$ is closed. Hence we can state the following:

THEOREM 3.1. *A viscous fluid spacetime obeying Einstein's equation with a cosmological constant is a connected non-compact Lorentzian* $[(PCRS)_4, \tilde{\nabla}]$ *with closed 1-form* α*, associated with the torsion tensor* τ*, corresponding to the semi-symmetric metric connection* $\tilde{\nabla}$*, the generator* μ *as the flow vector field of the fluid and the structure tensor* P *as the anisotropic pressure tensor of the fluid.*

By virtue of (3.3), (2.15) yields

$$\left(\frac{r}{2} + kp - \lambda - a\right) g + (k\sigma + kp + 6a - r)\gamma \otimes \gamma + (k - 2)P = 0. \tag{3.4}$$

From (3.4), we obtain by virtue of (2.7) that

$$\sigma = \frac{1}{2k}[3\tilde{r} - 2\lambda + 4a - 2(k - 2)b], \tag{3.5}$$

where $b = P(\mu, \mu)$. Again (3.4) yields by virtue of (2.7) that

$$p = \frac{1}{6k}[6\lambda - 3\tilde{r} - 2(k + 4)a - 2(k - 2)b]. \tag{3.6}$$

This leads to the following:

THEOREM 3.2. *In a viscous fluid* $[(PCRS)_4, \tilde{\nabla}]$ *spacetime, with closed 1-form* α*, associated with the torsion tensor* τ*, corresponding to the semi-symmetric metric connection* $\tilde{\nabla}$*, obeying Einstein's equation with a cosmolgical constant* λ *none of the isotropic pressure and energy density can be a constant.*

We assume that $p > 0$. Then since $\sigma > 0$, we have from (3.5) and (3.6) that

$$\lambda < \frac{3\tilde{r} + 4a - 2(k - 2)b}{2} \quad \text{and } \lambda > \frac{3\tilde{r} + 2(k + 4)a + 2(k - 2)b}{6}$$

and hence

$$\frac{3\tilde{r} + 2(k + 4)a + 2(k - 2)b}{6} < \lambda < \frac{3\tilde{r} + 4a - 2(k - 2)b}{2} \tag{3.7}$$

and

$$\tilde{r} > \frac{k-2}{3}(a+4b). \tag{3.8}$$

This leads to the following:

Theorem 3.3. *In a viscous fluid $[(PCRS)_4, \tilde{\nabla}]$ spacetime, with positive isotropic pressure and closed 1-form α, associated with the torsion tensor τ, corresponding to the semi-symmetric metric connection $\tilde{\nabla}$, obeying Einstein's equation, the cosmolgical constant λ satisfies the relation (3.7) and the scalar curvature $\tilde{r}$ satisfies the relation (3.8).*

We now investigate whether a viscous fluid $[(PCRS)_4, \tilde{\nabla}]$ spacetime with μ is the unit timelike flow vector field can admit heat flux or not. Therefore, if possible, let the energy-momentum tensor T be of the following form [**9**]

$$T(X,Y) = pg(X,Y) + (\sigma + p)\gamma(X)\gamma(Y) + \gamma(X)\eta(Y) + \gamma(Y)\eta(X),$$

where $\eta(X) = g(X,V)$ for all vector fields X; V being the heat flux vector field; σ, p are the energy density and isotropic pressure respectively. Thus we have $g(\mu, V) = 0$, i.e., $\eta(\mu) = 0$. Hence by virtue of the last relation, (2.7) and (2.15), (3.1) yields

$$\begin{aligned} 2P(X,Y) + \left(\lambda - \frac{\tilde{r}}{2} - kp - 2a\right) g(X,Y) + [\tilde{r} \\ -k(p+\sigma)]\gamma(X)\gamma(Y) - k[\gamma(X)\eta(Y) + \gamma(Y)\eta(X)] = 0. \end{aligned} \tag{3.9}$$

Setting $Y = \mu$ in (3.9), we obtain

$$2P(X,\mu) + \left(\lambda - \frac{3}{2}\tilde{r} + k\sigma - 2a\right)\gamma(X) + k\eta(X) = 0. \tag{3.10}$$

Putting $X = \mu$ in (3.10), we obtain

$$2b - \left(\lambda - \frac{3}{2}\tilde{r} + k\sigma - 2a\right) = 0.$$

Using the last relation in (3.10), we obtain

$$\eta(X) = -\frac{2}{k}[P(X,\mu) + b\gamma(X)], \quad \text{since } k \neq 0. \tag{3.11}$$

Thus we can state the following:

Theorem 3.4. *A viscous fluid $[(PCRS)_4, \tilde{\nabla}]$ spacetime, with closed 1-form α, associated with the torsion tensor τ, corresponding to the semi-symmetric metric connection $\tilde{\nabla}$, obeying Einstein's equation with a cosmolgical constant λ admits heat flux given by (3.11) unless $P(X,\mu) + b\gamma(X) \neq 0$ for all X.*

From (3.11), it follows that

$$V = -\frac{2}{k}(N + bI)\mu,$$

where $P(X,Y) = g(NX,Y)$ for all vector fields X, Y. This implies that $V = 0$ if and only if $-b$ is the eigenvalue of P corresponding to the eigenvector μ. Hence we can state the following:

Theorem 3.5. *A viscous fluid* $[(PCRS)_4, \tilde{\nabla}]$ *spacetime, with closed 1-form* α*, associated with the torsion tensor* τ*, corresponding to the semi-symmetric metric connection* $\tilde{\nabla}$*, can not admit heat flux if and only if* $-b$ *is the eigenvalue of* P *corresponding to the eigenvector* μ*.*

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