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ITERATIVE METHOD FOR FAMILIES OF NONEXPANSIVE MAPS

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Abstract. In this paper, we study an iterative method for approximating a common fixed point of a countably family of nonexpansive mappings in a Banach space. Our result is a generalization of the results of Bauschke [1], Maingé [12] and Chidume and Ali [4]. An asymptotic version of this class of operators is also studied and similar results obtained in a Banach space.

1. Introduction

Direct and iterative methods for finding fixed points of an operator defined in an appropriate Banach space have been studied by many authors. These studies have given rise to the development of results and techniques which are now widely available in the literature.

Existence and approximation results for a common fixed point of families of mappings have also been studied by various authors (see, for example [1, 2, 4, 6, 8, 9, 10, 11]).

Let $\{T_i\}$ be a countable family of nonexpansive mappings. We denote by a set $\mathcal{N}_I = \{i \in \mathrm{N} : T_i \neq I\}$ (I being the identity mapping on E). We also denote a set $\mathcal{F} = \cap_{i=1}^{\infty} F(T_i)$, the common fixed points of the operators $\{T_i,\ i = 1, 2, ...\}$. With these notations, Maingé [12] proved the following theorem:

Theorem 1.1. *Let K be a nonempty closed, convex subset of a real Hilbert space H. Let $\{T_i\}$ be a countable family of nonexpansive self mappings of K, $\{t_n\}$ and $\{\sigma_{i,t_n}\}$ be sequences in $(0,1)$ satisfying the following conditions: (i) $\lim\limits_{n\to\infty} t_n = 0$ (ii) $\sum_{i\geq 1} \sigma_{i,t_n} = (1 - t_n)$ (iii) $\forall i \in \mathcal{N}_I,\ \lim\limits_{n\to\infty} \frac{t_n}{\sigma_{i,t_n}} = 0$. Define a fixed point sequence $\{x_{t_n}\}$ by*

$$x_{t_n} = t_n C x_{t_n} + \sum_{i\geq 1} \sigma_{i,t_n} T_i x_{t_n}$$

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where $C: K \to K$ is a strict contraction. Assume $\mathcal{F} = \cap_{i=1}^{\infty} F(T_i) \neq \emptyset$, then $\{x_{t_n}\}$ converges strongly to a unique fixed point of the contraction $P_F o C$, where P_F is the metric projection from H onto F.

Chidume and Ali [4] considered an iterative scheme for approximating a common fixed point of a countable infinite family of nonexpansive mappings in a q-uniformly smooth Banach space and proved the following theorem:

Theorem 1.2. *Let E be a q-uniformly smooth Banach space with $q \geq 1 + d_q$. Let K be a closed, convex and nonempty subset of E. Let $\{t_n\}$ be a sequence in $(0,1)$ such that $\lim_{n\to\infty} t_n = 0$ and $\lim_{n\to\infty} \frac{t_n}{\sigma_{i,n}} = 0$ $\forall i \in \mathcal{N}_I$. Let $\{z_{t_n}\}$ be a sequence satisfying the equation $z_t = tu + \sum_{i\geq 1} \sigma_{i,t} S_i z_t$, and let $\mathcal{F} = \cap_{i=1}^{\infty} F(T_i) \neq \emptyset$. If the duality map j_q of E is weakly sequentially continuous, then z_{t_n} converges strongly to an element in $\mathcal{F}$.*

The above result of Chidume and Ali is a further generalization of the earlier results of Bauschke [1] and Maingé [12].

It is our purpose in this paper to show that the result of Chidume and Ali, theorem 1.2 of [4] (using exactly the same condition as in that paper) holds in an arbitrary Banach space. Furthermore, we prove a similar result for a countable family of asymptotically nonexpansive mappings, thus generalizing the class of operators as well.

2. Preliminaries

Let E be a real normed linear space with dual E^*. We denote by J the normalilzed duality mapping from E to 2^{E^*} defined by

$$Jx = \{f \in E^* : \langle x, f\rangle = ||x||^2 = ||f||^2\},$$

where $\langle ., .\rangle$ denotes the generalized duality pairing.It is well known that if E^* is strictly convex then J is single valued and if E^* is uniformly convex (equivalently if E is uniformly smooth) then J is uniformly continuous on bounded subsets of E. We shall denote the single-valued duality mapping by j.

A mapping T with domain $D(T)$ and range $R(T)$ in E is said to be *demiclosed* at p if when $\{x_n\}$ is a sequence in $D(T)$ such that $x_n \rightharpoonup x^* \in D(T)$ and $Tx_n \to p$ then $Tx^* = p$.
A Banach space E is called an Opial space[13] if for all sequence $\{x_n\} \in E$ such that $x_n \rightharpoonup x \in E$ the inequality

$$\liminf_{n\to\infty} ||x_n - x|| < \liminf_{n\to\infty} ||x_n - y||$$

holds for all $y \neq x$. For a normed linear space E, the existence of weakly sequentially continuous duality map implies E is an Opial space (see, for example [13]). If E is an Opial space and T is a nonexpansive map defined on E then $(I - T)$ is demiclosed at 0 (see, for example [13]).

In 1972, Goebel and Kirk [7] introduced a class of mappings generalizing the class of nonexpansive operators. Let K be a subset of a normed linear space. A mapping $T : K \to K$ is called asymptotically nonexpansive if there exists a sequence k_n, $k_n \geq 1$, such that $\lim_{n\to\infty} k_n = 1$ and $||T^n x - T^n y|| \leq k_n ||x - y||$ for each $x,\ y \in K$ and for each integer $n \geq 1$. For an example to show that the class of asymptotically nonexpansive maps contains properly the class of nonexpansive maps, see Goebel and Kirk [7].

Lemma 2.1. *Let E be a real normed linear space and let J denote the normalized duality map on E. Then for any given $x, y \in E$, the following inequality holds:*

$$||x+y||^2 \leq ||x||^2 + 2\langle y, j(x+y)\rangle \ \ \forall j(x+y) \in J(x+y).$$

3. Main Results

Let K be a nonempty closed and convex subset of a real Banach space E. Let $\{T_i\}_{i=1}^{\infty}$ be a family of nonexpansive self mappings of K. For a fixed $\delta \in (0,1)$, define a family of mappings $S_i : K \to K$ by $S_i x = (1-\delta)x + \delta T_i x \ \forall x \in K$, and $i \in \mathrm{N}$. For $t \in (0,1)$, let $\{\sigma_{i,t}\}_{i=1}^{\infty}$ be a sequence in $(0,1)$ such that $\sum_{i\geq 1} \sigma_{i,t} = (1-t)$. For arbitrary fixed $u \in K$,define a map $T_t : K \to K$ by

$$T_t x = tu + \sum_{i\geq 1} \sigma_{i,t} S_i x \ \forall x \in K \tag{1}$$

Then, T_t is a strict contraction on K. For if $x, y \in K$,

$$\begin{aligned}
||T_t x - T_t y|| &= ||\sum_{i\geq 1} \sigma_{i,t}\left((1-\delta)(x-y) + \delta(T_i x - T_i y)\right)|| \\
&\leq \sum_{i\geq 1} \sigma_{i,t}\left((1-\delta)||x-y|| + \delta||T_i x - T_i y||\right) \\
&\leq (1-t)\left((1-\delta)||x-y|| + \delta||x-y||\right) \\
&= (1-t)||x-y||.
\end{aligned}$$

Thus, for each $t \in (0,1)$, the map T_t is a strict contraction. Hence for each $t \in (0,1)$, T_t has a unique fixed point. Thus, for each $t \in (0,1)$, there exists a unique $z_t \in K$ satisfying

$$z_t = tu + \sum_{i\geq 1} \sigma_{i,t} S_i z_t \tag{2}$$

Lemma 3.1. *(see [4]) Let E be a real Banach space. Let K be a closed, convex and nonempty subset of E. For $t \in (0,1)$, let $\{z_t\}$ be a sequence satisfying 2 and assume $\mathcal{F} = \cap_{i=1}^{\infty} F(T_i) \neq \emptyset$. Then $\{z_t\}$ is bounded and admits a unique accumulation point as $t \to 0$*

Lemma 3.2. *Let E be a real Banach space. Let K be a closed, convex and nonempty subset of E. Let $\{t_n\}$ be a sequence in $(0,1)$ such that $\lim_{n\to\infty} t_n = 0$ and $\lim_{n\to\infty} \frac{t_n}{\sigma_{i,n}} = 0 \ \forall i \in \mathcal{N}_I$. Let $\{z_{t_n}\}$ be a sequence satisfying 2 and let $\mathcal{F} = \cap_{i=1}^{\infty} F(T_i) \neq \emptyset$. Then* $\lim_{n\to\infty} ||z_{t_n} - T_i z_{t_n}|| = 0 \ \forall i \in \mathrm{N}$.

Proof of Lemma 3.2 Since $u \in K$ is arbitrary, then for each $t \in (0,1)$ the sequence $T_t^n u$ converges to z_t. That is $T_t^n u \to z_t$ as $n \to \infty$. Thus, given $\epsilon_n > 0$, there exists N_{ϵ_n} such that $\forall m \geq N_{\epsilon_n}$, $||T_{t_n}^m u - z_{t_n}|| < \epsilon_n$. Hence, using Lemma 2.1, and for each $i \in$ N, we have:

$$\begin{aligned} ||S_i z_{t_n} - z_{t_n}||^2 &= ||S_i z_{t_n} - T_{t_n}^m u + T_{t_n}^m u - z_{t_n}||^2 \qquad m \geq N_{\epsilon_n} \\ &\leq ||T_{t_n}^m u - z_{t_n}||^2 + 2\left\langle S_i z_{t_n} - T_{t_n}^m u, j(S_i z_{t_n} - z_{t_n})\right\rangle \end{aligned}$$

$$\Rightarrow \frac{1}{2}||S_i z_{t_n} - z_{t_n}||^2 - \frac{1}{2}||T_{t_n}^m u - z_{t_n}||^2 \leq \left\langle S_i z_{t_n} - T_{t_n}^m u, j(S_i z_{t_n} - z_{t_n})\right\rangle. \tag{3}$$

Using 2 we have:

$$\left\langle z_{t_n} - T_{t_n}^m u, j(S_i z_{t_n} - z_{t_n})\right\rangle = t_n \left\langle u - T_{t_n}^m u, j(S_i z_{t_n} - z_{t_n})\right\rangle + \sum_{i\geq 1} \sigma_{i,n} \left\langle S_i z_{t_n} - T_{t_n}^m u, j(S_i z_{t_n} - z_{t_n})\right\rangle.$$

That is

$$\sum_{i\geq 1} \sigma_{i,n} \left\langle S_i z_{t_n} - T_{t_n}^m u, j(S_i z_{t_n} - z_{t_n})\right\rangle = \left\langle z_{t_n} - T_{t_n}^m u, j(S_i z_{t_n} - z_{t_n})\right\rangle - t_n \left\langle u - T_{t_n}^m u, j(S_i z_{t_n} - z_{t_n})\right\rangle.$$

Hence,

$$\begin{aligned} \left\langle S_i z_{t_n} - T_{t_n}^m u, j(S_i z_{t_n} - z_{t_n})\right\rangle &\leq (1-t)^{-1}||z_{t_n} - T_{t_n}^m u||||S_i z_{t_n} - z_{t_n}|| \\ &\quad - t_n(1-t_n)^{-1}\left\langle u - T_{t_n}^m u, j(S_i z_{t_n} - z_{t_n})\right\rangle \\ \leq (1-t_n)^{-1}\epsilon_n ||S_i z_{t_n} - z_{t_n}|| &\quad - t_n(1-t_n)^{-1}\left\langle u - T_{t_n}^m u, j(S_i z_{t_n} - z_{t_n})\right\rangle. \end{aligned}$$

Using this and 3, we have

$$\frac{1}{2}||S_i z_{t_n} - z_{t_n}||^2 - \frac{1}{2}||T_{t_n}^m u - z_{t_n}||^2 \leq (1-t_n)^{-1}\epsilon_n ||S_i z_{t_n} - z_{t_n}|| - t_n(1-t_n)^{-1}\left\langle u - T_{t_n}^m u, j(S_i z_{t_n} - z_{t_n})\right\rangle,$$

or

$$||S_i z_{t_n} - z_{t_n}||^2 \leq \epsilon_n^2 + 2\epsilon_n(1-t_n)^{-1}||S_i z_{t_n} - z_{t_n}|| - 2t_n(1-t_n)^{-1}\left\langle u - T_{t_n}^m u, j(S_i z_{t_n} - z_{t_n})\right\rangle.$$

Since $\{z_{t_n}\}$ is bounded and S_i is nonexpansive, we have that $\lim_{n\to\infty} ||S_i z_{t_n} - z_{t_n}|| = 0$. This implies $\lim_{n\to\infty} ||z_{t_n} - T_i z_{t_n}|| = \frac{1}{\delta} \lim_{n\to\infty} ||S_i z_{t_n} - z_{t_n}|| = 0$, thus completing the proof.

Theorem 3.3. *Let E be a real Banach space. Let K be a closed,convex and nonempty subset of E. Let $\{t_n\}$ be a sequence in $(0,1)$ such that $\lim_{n\to\infty} t_n = 0$ and $\lim_{n\to\infty} \frac{t_n}{\sigma_{i,n}} = 0$ $\forall i \in \mathcal{N}_I$. Let $\{z_{t_n}\}$ be a sequence satisfying 2 and let $\mathcal{F} = \cap_{i=1}^{\infty} F(T_i) \neq \emptyset$. If the duality mapping j of E is weakly sequentially continuous, then $\{z_{t_n}\}$ converges strongly to an element in $\mathcal{F}$.*

Proof Since $\{z_{t_n}\}$ is bounded, there exists a subsequence $\{z_{t_{n_k}}\}$ of $\{z_{t_n}\}$ that converges weakly to some $z \in K$. Using the demiclosedness property of $(I - T_i)$ for each $i \in \mathrm{N}$, and the fact that $\lim_{n\to\infty} ||z_{t_{n_k}} - T_i z_{t_{n_k}}|| = 0$, we get that $z \in \mathcal{F}$. Using 2 we have

$$\begin{aligned}
||z_{t_{n_k}} - z||^2 &= \left\langle z_{t_{n_k}} - z, j(z_{t_{n_k}} - z)\right\rangle \\
&= \left\langle t_{n_k}(u-z) + \sum_{i\geq 1} \sigma_{i,n_k}(S_i z_{t_{n_k}} - z), j(z_{t_{n_k}} - z)\right\rangle \\
&= \left\langle t_{n_k}(u-z), j(z_{t_{n_k}} - z)\right\rangle + \sum_{i\geq 1} \sigma_{i,n_k} \left\langle (S_i z_{t_{n_k}} - Sz), j(z_{t_{n_k}} - z)\right\rangle \\
&\leq \left\langle t_{n_k}(u-z), j(z_{t_{n_k}} - z)\right\rangle + \sum_{i\geq 1} \sigma_{i,n_k} ||z_{t_{n_k}} - z||^2.
\end{aligned}$$

The rest of the proof follows as in the proof of theorem 3.3 of Chidume and Ali [4], to establish that z_{t_n} converges strongly to z.

Corollary 3.4. *Let E be a compact real Banach space. Let K be a closed, convex and nonempty subset of E. Let t_n be a sequence in $(0,1)$ such that $\lim_{n\to\infty} t_n = 0$ and $\lim_{n\to\infty} \frac{t_n}{\sigma_{i,n}} = 0\ \forall i \in \mathcal{N}_I$. Let $\{z_{t_n}\}$ be a sequence satisfying 2 and let $\mathcal{F} = \cap_{i=1}^{\infty} F(T_i)$. Then $\{z_{t_n}\}$ converges strongly to an element of $\mathcal{F}$.*

Proof As K is a closed subset of a compact space, K is itself compact. It follows that the sequence $\{z_{t_n}\}$ has a subsequence $\{z_{t_{n_k}}\}$ which converges strongly to some $z \in K$. Compactness of E implies that E automatically admits a weak sequentially continuous duality map. The rest of the proof follows as in the proof of theorem 3.3

Let K be a nonempty closed and convex subset of a real Banach space E. Let $\{T_i\}$ be a family of asymptotically nonexpansive self mappings of K. For a fixed $\delta \in (0,1)$, define a family of mappings $S_i : K \to K$ by $S_i x = (1-\delta)x + \delta T_i^m x\ \forall x \in K$ and $i \in \mathrm{N}$. Let $\{k_n\}$ be the sequence appearing in the definition of asymptotically nonexpansive maps and let $M = \sup_n k_n$. Let $d_0 \in (0,1)$ be such that $(1-t)(1-\delta+\delta M) < 1\ \forall t \in (d_0, 1)$. Let $\sum_{i\geq 1} \sigma_{i,t} = (1-t)$. For arbitrary fixed $u \in K$, define a map $T_t : K \to K$ by

$$T_t x = tu + \sum_{i\geq 1} \sigma_{i,t} S_i x\ \forall x \in K.$$

Then T_t is a strict contraction on K. For if $x, y \in K$, we have

$$\begin{aligned}
||T_t x - T_t y|| &= ||\sum_{i\geq 1} \sigma_{i,t}\left((1-\delta)(x-y) + \delta(T_i^m x - T_i^m y)\right)|| \\
&\leq (1-t)\left((1-\delta)||x-y|| + \delta k_n ||x-y||\right) \\
&\leq (1-t)(1-\delta+\delta M)||x-y|| \\
&< \alpha||x-y||, \qquad \alpha \in (0,1).
\end{aligned}$$

Thus for each $t \in (0,1)$, there is a unique $z_t \in K$ such that

$$z_t = tu + \sum_{i \geq 1} \sigma_{i,t} S_i z_t \tag{4}$$

Lemma 3.5. *Let E be a real Banach space. Let K be a closed, convex and nonempty subset of E. For $t \in (d_0, 1)$, let z_t be a sequence satisfying 4 and assume $\mathcal{F} = \cap_{i=1}^{\infty} \neq \emptyset$. Then $\{z_t\}$ is bounded and admits a unique accumulation point as $t \to 0$.*

Proof Let $x* \in \mathcal{F}$. Using 4, we have

$$\begin{aligned}
||z_t - x^*||^2 &= \left\langle t(u - x^*) + \sum_{i \geq 1} \sigma_{i,t}(S_i z_t - x^*), j(z_t - x^*) \right\rangle \\
&\leq t \langle u - x^*, j(z_t - x^*) \rangle + \sum_{i \geq 1} \sigma_{i,t} \alpha_n ||z_t - x^*||^2, \ \alpha_n = 1 - \delta + \delta k_n \\
&= t \langle u - x^*, j(z_t - x^*) \rangle + (1-t)\alpha_n ||z_t - x^*||^2 \\
&\Rightarrow ||z_t - x^*||^2 \leq t \langle u - x^*, j(z_t - x^*) \rangle + (1-t)||z_t - x^*||^2 \\
&\Rightarrow ||z_t - x^*|| \leq ||u - x^*||
\end{aligned}$$

Thus $\{z_t\}$ is bounded.

We now show that $\{z_t\}$ has a unique accumulation point as $t \to d_0$. Assume for contradiction that $\{z_t\}$ admits two distinct accumulation points x^* and y^*. Then,

$$\begin{aligned}
||x^* - y^*||^2 &\leq \left\langle t(u - x^*) + \sum_{i \geq 1} \sigma_{i,t}(S_i y^* - x^*), j(y^* - x^*) \right\rangle, \\
&\leq t \langle u - x^*, j(y^* - x^*) \rangle + \sum_{i \geq 1} \sigma_{i,t} \alpha_n ||y^* - x^*||^2 \\
&\leq t \langle u - x^*, j(y^* - x^*) \rangle + \sum_{i \geq 1} \sigma_{i,t} ||y^* - x^*||^2 \\
&= t \langle u - x^*, j(y^* - x^*) \rangle + (1-t)||y^* - x^*||^2, \\
||y^* - x^*||^2 &\leq \langle u - x^*, j(y^* - x^*) \rangle .
\end{aligned}$$

Similarly,

$$||x^* - y^*||^2 \leq \langle u - y^*, j(x^* - y^*) \rangle .$$

Hence,

$$2||x^* - y^*||^2 \leq ||x^* - y^*||^2,$$

contradiction. This contradiction shows that $x^* = y^*$.

By mimicking the proof of Lemma 3.2, it is easy to establish the following Lemma:

Lemma 3.6. *Let E be a Banach space. Let K be a closed, convex and nonempty subset of E. Let $\{t_n\}$ be a sequence in $(d_0, 1)$ such that $\lim_{n \to \infty} t_n = 0$ and $\lim_{n \to \infty} \frac{t_n}{\sigma_{i,n}} = 0 \ \forall i \in \mathcal{N}_I$. Let $\{z_{t_n}\}$ be a sequence satisfying 4. Let $\{T_i\}$ be a family of asymptotically* *nonexpansive self mappings of* K *and let* $\mathcal{F} = \cap_{i=1}^{\infty} F(T_i) \neq \emptyset$. *Then* $\lim_{n\to\infty} ||z_{t_n} - T_i z_{t_n}|| = 0 \ \forall i \in \mathrm{N}$.

Also, by mimicking the proof of theorem 3.3, it is easy to establish the following theorem:

Theorem 3.7. *Let* E *be a Banach space, and let* K *be a closed, convex and nonempty subset of* E. *Let* $\{t_n\}$ *be a sequence in* $(d_0, 1)$ *such that* $\lim_{n\to\infty} t_n = 0$ *and* $\lim_{n\to\infty} \frac{t_n}{\sigma_{i,n}} = 0 \ \forall i \in \mathcal{N}_I$. *Let* $\{z_{t_n}\}$ *be a sequence satisfying 4. Let* $\{T_i\}$ *be a countable family of asymptotically nonexpansive self mappings of* K *such that* $\mathcal{F} = \cap_{i=1}^{\infty} F(T_i) \neq \emptyset$. *If* E *admits a weakly continuous duality map, then* $\{z_{t_n}\}$ *converges strongly to some* $z \in \mathcal{F}$.

References

[1] H. H. Bauschke, The approximations of fixed points of compositions of nonexpansive mappings in Hilbert space, J. Math. Anal. Appl. 202 (1996), 150-159.

[2] L. P. Belluce and W. A. Kirk, Fixed point theorem for families of contraction mapping, Pacific J. Math. 18 (1966), 213-217.

[3] J. V. Caristi, Fixed point theorems for mappings satisfying inward conditions, Trans. Amer. Math. Soc. 215 (1976), 241-251.

[4] C. E. Chidume and Bashir Ali, Convergence of path and an iterative method for families of nonexpansive mappings, Applicable Analysis 87 (2008), 117-129.

[5] C. E. Chidume and S. J. Aneke, Existence, uniqueness and approximation of a solution for a K-positive definite operator equation, Appl. Anal. 50 (1993), 285–294.

[6] R. De Marr, Common fixed points for commuting contraction mappings, Pacific J. Math. 13 (1963), 1139-1141.

[7] K. Goebel and W. A. Kirk, A fixed point theorem for asymptotically nonexpansive mappings, Proc. Amer. Math. Soc. 35 (1972), 171 –174.

[8] J. S. Jung, Iterative approaches to common fixed points of nonexpansive mappings in Banach spaces, J. Math. Anal. Appl. 302 (2005), 509-520.

[9] J. S. Jung, Y. J. Cho, R. P. Agarwal, Iterative schemes with some control conditions for finite family of nonexpansive mappings in Banach spaces, Fixed point theory and Appl.2 (2005), 125-135.

[10] S. Kakutani, Two fixed point theorems concerning bicompact convex sets, Proc. Imp. Acad. Tokyo 14 (1938), 242-245.

[11] T. C. Lim, A fixed point theorem for families of nonexpansive mappings, Pacific. J. Math. 53 (1974), 487-493.

[12] P. Maingé, Approximation methods for common fixed points of nonexpansive mappings in Hilbert space, J. Math. Anal. Appl. 325 (2007), 469-479.

[13] Z. Opial, Weak convergence of the sequence of successive approximations for nonexpansive mappings, Bull. Amer. Math. Soc. 73 (1967), 591-597.

[14] T. Suzuki, Strong convergence of Krasnoselskii and Mann's type sequences for one-parameter nonexpansive semigroups without Bochner integrals, J. Math. Anal. Appl. 305, (2005), 227-239.

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