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Three dimensional generalized Weyl fractional calculus pertaining to three-dimensional $\bar{H}$-Transform

V. B. L. Chaurasia* and Monika Jain**

Abstract. The aim of this paper is to evaluate certain triple integral relations involving H-function and the multivariable H-function. Next, we give three Theorems containing the product of H-function, with the help of our main findings and using the Mellin integral transform. The results obtained here are quite general in nature due to the presence of functions which are basic in nature. A large number of new results have been obtained by proper choice of parameter.

1. Introduction

In this paper, we have made an attempt to derive a theorem on three-dimensional $\bar{H}$-transform having Weyl type three dimensional operators. The results obtained here are basic in nature and include the results given earlier by Saigo, Saxena and Ram [14], Saxena and Ram [17], Chaurasia and Srivastava [3], etc. A few interesting and elegant results as special cases of our main results has also been recorded.

2. Fractional Integrals and derivatives

Useful and interesting generalization of both the Riemann-Liouville and Erdlyi-Kober fractional integration operators is introduced by Saigo [10], [11] in terms of Gauss's hypergeometric function as given below.

Let α, β and η are complex numbers and let y $\in R_+$ $(0, \infty)$. Following [10], [11] the fractional integral $(Re(\alpha) > 0)$ and derivative $(Re(\alpha) < 0)$ of the first kind of a function f (y) on R_+ are defined respectively in the forms

(2.1)
$$I_{0,y}^{\alpha,\beta.\eta} f = \frac{y^{-\alpha-\beta}}{\Gamma(\alpha)} \int_0^y (y-t)^{\alpha-1} {}_2F_1\left(\alpha+\beta, -\eta\,;\, \alpha\,;\, 1-\frac{t}{y}\right) f(t)dt\,; Re(\alpha) > 0$$

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$$(2.2) \qquad = \frac{d^n}{dy^n} I_{0,y}^{\alpha+n,\beta-n,\eta-n} f, 0 < Re(\alpha) + n \leqq 1 \ (n = 1, 2, ...)$$

where ${}_2F_1$ (a, b; c; .) is Gauss's hypergeometric function. The fractional integral ($Re(\alpha) > 0$) and derivative ($Re(\alpha) < 0$) of the second kind are given by

(2.3)

$$J_{y,\infty}^{\alpha,\beta,\eta} f = \frac{1}{\Gamma(\alpha)} \int_y^\infty (t-y)^{\alpha-1} t^{-\alpha-\beta} {}_2F_1 \left(\alpha+\beta, -\eta; \alpha; 1 - \frac{y}{t}\right) f(t)dt, Re(\alpha) > 0$$

$$(2.4) \qquad = (-1)^n \frac{d^n}{dy^n} I_{y,\infty}^{\alpha+n,\beta-n,\eta} f, 0 < Re(\alpha) + n \leqq 1 \ (n = 1, 2, ...)$$

The Riemann-Liouville, Weyl and Erdlyi-Kober fractional calculus operators follow as special cases of the operators I and J as given below

$$(2.5) \qquad R_{0,y}^{\alpha} f = I_{0,y}^{\alpha,-\alpha,\eta} f = \frac{1}{\Gamma(\alpha)} \int_0^y (y-t)^{\alpha-1} f(t)dt, Re(\alpha) > 0$$

$$(2.6) \qquad = \frac{d^n}{dy^n} R_{0,y}^{\alpha+n} f, 0 < Re(\alpha) + n \leqq 1 \ (n = 1, 2, ...)$$

$$(2.7) \qquad W_{y,\infty}^{\alpha} f = J_{y,\infty}^{\alpha,-\alpha,\eta} f = \frac{1}{\Gamma(\alpha)} \int_y^\infty (t-y)^{\alpha-1} f(t)dt, Re(\alpha) > 0$$

$$(2.8) \qquad = (-1)^n \frac{d^n}{dy^n} W_{y,\infty}^{\alpha+n} f, 0 < Re(\alpha) + n \leqq 1 \ (n = 1, 2, ...)$$

$$(2.9) \qquad E_{0,y}^{\alpha,\eta} f = I_{0,y}^{\alpha,0,\eta} f = \frac{y^{-\alpha-\eta}}{\Gamma(\alpha)} \int_0^y (y-t)^{\alpha-1} t^{\eta} f(t)dt, \ Re(\alpha) > 0$$

$$(2.10) \qquad K_{y,\infty}^{\alpha,\eta} f = J_{y,\infty}^{\alpha,0,\eta} f = \frac{y^{\eta}}{\Gamma(\alpha)} \int_y^\infty (t-y)^{\alpha-1} t^{-\alpha-\eta} f(t)dy, Re(\alpha) > 0$$

Following Miller [9, p.82], we denote by u_1 the class of functions f (x) on R_+ which are infinitely differentiable with partial derivatives of any order behaving as $0\,(\,|\,x|^{-\xi})$when x $\rightarrow \infty$ for all ξ. Similarly by u_2, we denote the class of functions f (x,y) on R_+ x R_+, which are infinitely differentiable with partial derivatives of any order behaving as $0\,(\,|\,x|^{-\xi_1}\,|\,y|^{-\xi_2})$when x $\rightarrow \infty$, y $\rightarrow \infty$ for all ξ_i (i = 1,2).

On the same pattern by u_3, we denote the class of functions $F(x, y, z)$ on R_+ x R_+ x R_+, which are infinitely differentiable with partial derivatives of any order behaving as $0\,(\,|\,x|^{-\xi_1}\,|\,y|^{-\xi_2}\,|\,z|^{-\xi_2})$ when x $\rightarrow \infty$, y $\rightarrow \infty$, z $\rightarrow \infty$ for all ξ_i (i = 1,2,3).

The three dimensional operator of Weyl type fractional integration of orders $Re(a) > 0$, $Re(d) > 0$, $Re(g) > 0$ is defined in the class u_3 by

$$J_{p,\infty}^{a,b,c} J_{q,\infty}^{d,e,f} J_{r,\infty}^{g,h,k} [f(p,q,r)]$$

$$= \frac{p^b q^e r^h}{\Gamma(a)\Gamma(d)\Gamma(g)} \int_p^\infty \int_q^\infty \int_r^\infty (u-p)^{a-1} (v-q)^{d-1} (w-r)^{g-1} u^{-a-b} v^{-d-e} w^{-g-h}$$

(2.11)

$$. {}_2F_1 \left(a+b, -c; a; 1-\frac{p}{u}\right) {}_2F_1 \left(d+e, -f; d; 1-\frac{q}{v}\right) {}_2F_1 \left(g+h, -k; g; 1-\frac{r}{w}\right) .f(u,v,w) du dv dw,$$

where b, e, h, c, f, k are real numbers. More generally, the operator [11] of Weyl type fractional calculus in three variables is defined by the differ-integral expression as

$$J_{p,\infty}^{a,b,c} J_{q,\infty}^{d,e,f} J_{r,\infty}^{g,h,k} [f(p,q,r)] = \frac{(-1)^{r_1+r_2+r_3} p^b q^e r^h}{\Gamma(a+r_1)\Gamma(d+r_2)\Gamma(g+r_3)} \frac{\partial^{r_1+r_2+r_3}}{\partial p^{r_1} \partial q^{r_2} \partial r^{r_3}} \tag{2.12}$$

$$\left\{ \int_p^\infty \int_q^\infty \int_r^\infty (u-p)^{a+r_1-1} (v-q)^{d+r_2-1} (w-r)^{g+r_3-1} u^{-a-b} v^{-d-e} w^{-g-h} \right.$$

$$. {}_2F_1 \left(a+b, -c; a; 1-\frac{p}{u}\right) {}_2F_1 \left(d+e, -f; d; 1-\frac{q}{v}\right) {}_2F_1 \left(g+h, -k; g; 1-\frac{r}{w}\right)$$

$$\left. . \quad f(u,v,w) du dv dw \right\}$$

for arbitrary real (complex) a, d and g, r_1, r_2, r_3 = 0,1,. . .
In particular, if $Re(a) < 0$, $Re(g) < 0$ and r_1, r_2, r_3 are positive integers such that $Re(a) + r_1 > 0$, $Re(d) + r_2 > 0$, $Re(g) + r_3 > 0$, then (2.12) yields the partial fractional derivative of f (p, q, r).
On the other hand if we set b = e = h = 0, (2.12) yields the Weyl type operators in three dimensions as

$$K_{p,\infty}^{a,c} K_{q,\infty}^{d,f} K_{r,\infty}^{g,k} [f(p,q,r)] = J_{p,\infty}^{a,0,c} J_{q,\infty}^{d,0,f} J_{r,\infty}^{g,0,f} [f(p,q,r)] \tag{2.13}$$

$$= \frac{(-1)^{r_1+r_2+r_3} p^c q^f r^k}{\Gamma(a+r_1)\Gamma(d+r_2)\Gamma(g+r_3)} \frac{\partial^{r_1+r_2+r_3}}{\partial p^{r_1} \partial q^{r_2} \partial r^{r_3}} \left\{ \int_p^\infty \int_q^\infty \int_r^\infty (u-p)^{a+r_1-1} \right.$$

$$\left. . \ (v-q)^{d+r_2-1} (w-r)^{g+r_3-1} u^{-a-b} v^{-d-e} w^{-g-h} f(u,v,w) du dv dw \right\}$$

3. Three-dimensional Laplace and $\bar{H}$-Transforms

The Laplace transform ζ (p,q,r) of a function $F(x,y,z) \in u_3$ is defined as

$$\zeta\,(p,q,r) = \alpha\,[\,f(x,y,z);p,q,r] = \int_0^\infty \int_0^\infty \int_0^\infty e^{-px-qy-rz} f(x,y,z)dxdydz \tag{3.1}$$

where Re (p) ¿ 0, Re (q) ¿ 0, Re (r) ¿ 0. Similarly, the Laplace transform of

$$f[u\sqrt{x^2-\,\lambda_1^2}\,H(x-\lambda_1),\; v\sqrt{y^2-\,\lambda_2^2}\,H(y-\lambda_2),\; w\sqrt{z^2-\,\lambda_3^2}\,H(z-\lambda_3)\,]$$

is defined by the Laplace transform of $F(x,y,z)$ where

$$F(x,y,z) = f[u\sqrt{x^2-\lambda_1^2}H(x-\lambda_1),\; v\sqrt{y^2-\lambda_2^2}\,H(t-\lambda_2),\; w\sqrt{z^2-\lambda_3^2}\,H(z-\lambda_3)],$$

$$x\,>\lambda_1>0,\;\; y>\lambda_2\,>\,0,\;\; z\,>\,\lambda_3\,>\,0 \tag{3.2}$$

and H (t) denotes Heaviside's unit step function.

Definition. By three dimensional $\bar{H}$-transform ϕ (p,q,r) of a function $F(x,y,z)$, we mean the following repeated integral involving three different $\bar{H}$-functions

$$\phi\,(p,q,r) = \phi^{M_1,N_1;\,M_2,N_2;\,M_3,N_3}_{P_1,Q_1\,;\,P_2,Q_2\,;\,P_3,Q_3}\,[F(x,y,z);\alpha,\;\beta,\;\gamma\,;\,p,q,r]$$

$$= \int_{\lambda_1}^\infty \int_{\lambda_2}^\infty \int_{\lambda_3}^\infty (px)^{\alpha-1}\,(qy)^{\beta-1}\,(rz)^{\gamma-1}$$

$$.\,\bar{H}^{M_1,N_1}_{P_1,Q_1}\left[(px)^{k_1}\,\middle|\,\begin{matrix}(a_j,\alpha_j;A_j)_{1,N_1},(a_j,\alpha_j)_{N_1+1,P_1}\\(b_j,\beta_j)_{1,M_1},(b_j,\beta_j;B_j)_{M_1+1,Q_1}\end{matrix}\right]$$

$$.\,\bar{H}^{M_2,N_2}_{P_2,Q_2}\left[(qy)^{k_2}\,\middle|\,\begin{matrix}(c_j,\kappa_j;c_j)_{1,N_2},(c_j,\kappa_j)_{N_2+1.P_2}\\(d_j,\tau_j)_{1,M_2},(d_j,\tau_j;D_j)_{M_2+1.Q_2}\end{matrix}\right]$$

$$.\,\bar{H}^{M_3,N_3}_{P_3,Q_3}\left[(rz)^{k_3}\,\middle|\,\begin{matrix}(e_j,\rho_j;E_j)_{1,N_3},(e_j,\rho_j)_{N_3+1.P_3}\\(f_j,\psi_j)_{1,M_3},(f_j,\psi_j;F_j)_{M_3+1.Q_3}\end{matrix}\right] f(x,y,z)dxdydz \tag{3.3}$$

Here we suppose that $\lambda_1\,>\,0,\,\lambda_2\,>\,0,\;\lambda_3\,>\,0,\,k_1\,>\,0,\,k_2\,>\,0,\,k_3\,>\,0\,;\,\phi\,(p,q,r)$exists and belongs to u_3. Further suppose that

$$|\,arg p^{k_1}|\;<\;\frac{1}{2}\,T_1\pi,\;\;|\,arg q^{k_2}|\;<\;\frac{1}{2}\,T_2\pi,\;|\,arg r^{k_3}|\;<\;\frac{1}{2}\,T_3\,\pi \tag{3.4}$$

where

$$T_1 = \sum_{j=1}^{M_1}|\,\beta_j| + \sum_{j=1}^{N_1} A_j\,a_j - \sum_{M_1+1}^{Q_1}|\,B_j\,\beta_j| \;-\; \sum_{j=N_1+1}^{P_1}\alpha_j\;>\;0$$

$$T_2 = \sum_{j=1}^{M_2}|\,\tau_j|\;+\;\sum_{j=1}^{N_2} C_j c_j - \sum_{M_2+1}^{Q_2}|\,D_j\tau_j|\;-\;\sum_{j=N_2+1}^{P_2}\kappa_j\;>\;0$$

$$T_3 \;=\; \sum_{j=1}^{M_3} |\,\psi_j| \,+\, \sum_{j=1}^{N_3} E_j\, e_j - \sum_{M_3+1}^{Q_3} |\, f_j\, \psi_j| \;-\; \sum_{j=N_3+1}^{P_3} \rho_j \;>\; 0$$

The $\bar{H}$-function appearing in (3.3), introduced by Inayat-Hussain ([6], see also [2]) in terms of Mellin-Barnes type contour integral, is defined by

$$\bar{H}^{M,N}_{P,Q}\left[z\left|{}^{(a_j,\alpha_j;A_j)_{1,N}\,,\,(a_j,\alpha_j)_{N+1,P}}_{(b_j,\beta_j)_{1,M}\,,\,(b_j,\beta_j;\,B_j)_{M+1,Q}}\right.\right] \;=\; \frac{1}{2\pi i}\int_{-i\infty}^{i\infty}\psi\,(\xi)\,z^{\xi}\,d\xi \tag{3.5}$$

where

$$\psi\,(\xi) = \frac{\prod_{j=1}^{M}\Gamma(b_j-\beta_j\xi)\,\prod_{j=1}^{N}\{\Gamma(1-a_j+\alpha_j\xi)\}^{A_j}}{\prod_{j=M+1}^{Q}\{\Gamma(1-b_j+\beta_j\xi)^{B_j}\,\prod_{j=N+1}^{P}\Gamma(a_j-\alpha_j\xi)} \tag{3.6}$$

which contains fractional powers of some of the Γ-functions. Here and throughout the paper A_j (j = 1,. . . ,P) and B_j (j = 1,. . . ,Q) are complex parameters, αj $\geqslant 0$ (j=1,. . . ,P), βj $\geqslant 0$ (j = 1,. . . ,Q), (not all zero simultaneously) and the exponents A_j (j = 1,. . . ,N) and B_j (j = M+1,. . . ,Q) can take on non-integer values. The contour in (3.5) is imaginary axis Re $(\xi) = 0$. It is suitably indented in order to avoid the singularities of the Γ-functions and to keep these singularities on appropriate sides. Again, for A_j (j = 1,. . . ,N) not an integer, the poles of the Γ-functions of the numerator in (3.6) are converted to branch points. However, as long as there is no coincidence of poles from any $\Gamma(b_j-\beta_j\xi),(j=1,...,M) and \Gamma(1-a_j+\alpha_j\xi)$, (j= 1,. . . ,N) pair the branch cuts can be chosen so that the path of integration can be distorted in the usual manner.

For the sake of brevity

$$T = \sum_{j=1}^{M} |\,\beta_j\,| \;+\; \sum_{j=1}^{N} A_j\alpha_j - \sum_{j=M+1}^{Q} |\,B_j\,\beta_j| \;-\; \sum_{j=N+1}^{P} \alpha_j \;>\; 0 \tag{3.7}$$

4. Relationship Between Three-Dimensional $\bar{H}$-transforms in Terms of Three-Dimensional operator of Weyl Type

To prove the theorem in this section, we need the following three-dimensional $\bar{H}$-transform ϕ_1 (p,q,r) of $F(x,y,z)$ defined by

$$\phi_1(p,q,r) = \bar{H}^{M_1+1,N_1\;\,;\,M_2+1,N_2\;\,;\,M_3+1,N_3}_{P_1+1,Q_1+1\,;\,P_2+1,Q_2+1\,;\,P_3+1,Q_3+1}\,[F(x,y,z);\alpha,\beta,\gamma\,;\,p,q,r]$$

$$= \int_{\lambda_1}^{\infty}\int_{\lambda_2}^{\infty}\int_{\lambda_3}^{\infty}(px)^{\alpha-1}\,(qy)^{\beta-1}\,(rz)^{\gamma-1}$$

$$.\,\bar{H}^{M_1+2,N_1}_{P_1+2,Q_1+2}\left[(px)^{k_1}\left|{}^{(a_j,\alpha_j;A_j)_{1,N_1}\,,\,(a_j,\alpha_j)_{N_1+1,P_1}\,,\,(1-\alpha,k_1),\,(a+b+c-\alpha+1,k_1)}_{(b-\alpha+\alpha,k_1),\,(c-\alpha+1,k_1)\,,\,(b_j,\beta_j)_{1,M_1},(b_j,\beta_j)_{M_1+1,Q_1}}\right.\right]$$

$$. \bar{H}^{M_2+2,N_2}_{P_2+2,Q_2+2}\left[(qy)^{k_2}\left|\begin{smallmatrix}(c_j,\kappa_j;C_j)_{1,N_2},(c_j,\kappa_j)_{N_2+1,P_2},(1-\beta,k_2),(d+e+f-\beta+1,k_2)\\(e-\beta+1,k_1),(f-\beta+1,k_2),(d_j,\tau_j)_{1,M_2},(d_j,\tau_j;D_j)_{M_2+1,Q_2}\end{smallmatrix}\right.\right]$$

$$.\bar{H}^{M_3+2,N_3}_{P_3+2,Q_3+2}\left[(rz)^{k_3}\left|\begin{smallmatrix}(e_j,\rho_j;E_j)_{1,N_3},(e_j,\rho_j)_{N_3+1,P_3},(1-\gamma,k_3),(g+h+k-\gamma+1,k_3)\\(h-\gamma+1,k_3),(k-\gamma+1,k_3),(f_j,\psi_j)_{1,M_3},(f_j,\psi_j;F_j)_{M_3+1,Q_3}\end{smallmatrix}\right.\right] F(x,y,z)dxdydz,$$

where it is assumed that ϕ_1 (p,q,r) exists and belongs to u_3 as well as $k_1 > 0$, $k_2 > 0$, $k_3 > 0$ and other conditions on the parameters, in which additional parameters a, b, d, e, g, h, c, f, k included correspond to those in (2.11).

Theorem 1. Let ϕ (p,q,r) be given by definition (3.3) then for $Re(a) > 0$, $Re(d) > 0$, $Re(g)$¿ 0, $\lambda_1 > 0$, λ2 ¿ 0, λ3 ¿ 0, $k_1 > 0$, $k_2 > 0$, $k_3 > 0$ there holds the formula

$$J^{a,b,c}_{p,\infty}\, J^{d,e,f}_{q,\infty}\, J^{g,h,k}_{r,\infty}[\,\phi\,(p,q,r)] = \phi_1\,(p,q,r) \tag{4.1}$$

provided that ϕ_1 (p,q,r) exists and belong to u_3.

Proof: Let $Re(a) > 0$, $Re(d) > 0$, $Re(g) > 0$ then in view of (2.11) and (3.3), we find that

$$J^{a,b,c}_{p,\infty}\, J^{d,e,f}_{q,\infty}\, J^{g,h,k}_{r,\infty}[\,\phi\,(p,q,r)] = \frac{p^b q^e r^h}{\Gamma(a)\Gamma(d)\Gamma(g)}\int_p^\infty\int_q^\infty\int_r^\infty (u-p)^{a-1}\,(v-q)^{d-1}$$

$$.\,(w-r)^{g-1}\,u^{-a-b}\,v^{-d-e}\,w^{-g-h}\,{}_2F_1\left(a+b,-c;a;1-\frac{\beta}{u}\right)\,{}_2F_1\left(d+e,-f;d;1-\frac{q}{v}\right)$$

$$.\,{}_2F_1\left(g+h,-k;g;1-\frac{r}{w}\right)\,\phi\,(u,v,w)dudvdw$$

$$= \frac{p^b q^e r^h}{\Gamma(a)\Gamma(d)\Gamma(g)}\int_p^\infty\int_q^\infty\int_r^\infty u^{-a-b}v^{-d-e}\,w^{-g-h}\,(u-p)^{a-1}\,(v-q)^{d-1}$$

$$.(w-r)^{g-1}\,{}_2F_1\left(a+b,-c;a;1-\frac{p}{u}\right)\,{}_2F_1\left(d+e,-f;d;1-\frac{q}{v}\right)$$

$$.\,{}_2F_1\left(g+h,-k;g;1-\frac{r}{w}\right)\left\{\int_{\lambda_1}^\infty\int_{\lambda_2}^\infty\int_{\lambda_3}^\infty (ux)^{\alpha-1}\,(vy)^{\beta-1}\,(wz)^{\gamma-1}\right.$$

$$.\,\bar{H}^{M_1,N_1}_{P_1,Q_1}\left[(ux)^{k_1}\left|\begin{smallmatrix}(a_j,\alpha_j;A_j)_{1,N_1},(a_j,\alpha_j)_{N_1+1,P_1}\\(b_j,\beta_j)_{1,M_1},(b_j,\beta_j)_{M_1+1,Q_1}\end{smallmatrix}\right.\right]$$

$$.\,\bar{H}^{M_2,N_2}_{P_2,Q_2}\left[(vy)^{k_2}\left|\begin{smallmatrix}(c_j,\kappa_j;C_j)_{1,N_2},(c_j,\kappa_j)_{N_2+1,P_2}\\(d_j,\tau_j)_{1,M_2},(d_j,\tau_j;D_j)_{M_2+1,Q_2}\end{smallmatrix}\right.\right]$$

$$\left..\bar{H}^{M_3,N_3}_{P_3,Q_3}\left[(wz)^{k_3}\left|\begin{smallmatrix}(e_j,\rho_j;E_j)_{1,N_3},(e_j,\rho_j)_{N_3+1,P_3}\\(f_j,\psi_j)_{1,M_3},(f_j,\psi_j;F_j)_{M_3+1,Q_3}\end{smallmatrix}\right.\right] F(x,y,z)dxdydz,\right\}\,dudvdw. \tag{4.2}$$

On interchanging the order of integration which is permissible and on evaluating the u, v and w-integrals through the integral formula

$$\int_x^\infty u^{-\mu-\nu}(u-x)^{\nu-1}\,{}_2F_1\left(\tau,\,\omega\,;\,\nu\,;\,1-\frac{x}{u}\right)$$

$$.\,\bar{H}^{M,N}_{P,Q}\left[(au)^k\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,N},\,(a_j,\alpha_j)_{N+1,P}\\(b_j,\beta_j)_{1,M},(b_j,\beta_j)_{M+1,Q}\end{matrix}\right.\right]du$$

$$(4.3)\qquad=\frac{\Gamma(\nu)}{x^\mu}\,\bar{H}^{M+2,N}_{P+2,Q+2}\left[(ax)^k\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,N},\,(a_j,\alpha_j)_{N+1,P},(\mu+\nu-\tau,k;1),(\mu+\nu-\omega,k;1)\\(\mu,k;1),(\mu+\nu-\tau-\omega,k;1),\,(b_j,\beta_j)_{1,M},(b_j,\beta_j)_{M+1,Q}\end{matrix}\right.\right],$$

where $Re(\nu)\;>\;0,\;Re\left(\mu+\nu+\;\frac{k(1-a_j)}{\alpha_j}\right)\;>\;0$

$$Re\left(\mu+\nu-\tau-\omega+\;\frac{k(1-a_j)}{\alpha_j}\right)\;>\;0,\;|argz|<\frac{1}{2}T\pi\quad(Tisgivenin(20))$$

(4.3) can be established by means of the following formula [4, p.399].

$$\int_0^1 x^{\gamma-1}(1-x)^{\rho-1}\,{}_2F_1\,(\alpha,\beta;\,\gamma;x)dx=\;\frac{\Gamma(\gamma)\,\Gamma(\rho)\,\Gamma(\gamma+\rho-\alpha-\beta)}{\Gamma(\gamma+\rho-\alpha)\,\Gamma(\gamma+\rho-\beta)}$$

For Re (γ) ¿ 0, Re (ρ) ¿ 0, Re (γ +ρ - α- β) ¿ 0.
The left hand side of (4.3) becomes

$$=\;\int_{\lambda_1}^\infty\int_{\lambda_2}^\infty\int_{\lambda_3}^\infty(px)^{\alpha-1}\,(qy)^{\beta-1}\,(rz)^{\gamma-1}$$

$$.\,\bar{H}^{M_1+2,N_1}_{P_1+2,Q_1+2}\left[(px)^{k_1}\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,N_1},\,(a_j,\alpha_j)_{N_1+1,P_1},\,(1-\alpha,k_1),\,(a+b+c-\alpha+1,k_1)\\(b-\alpha+1,k_1),\,(c-\alpha+1,k_1)\,,\,(b_j,\beta_j)_{1,M_1},(b_j,\beta_j)_{M_1+1,Q_1}\end{matrix}\right.\right]$$

$$.\,\bar{H}^{M_2+2,N_2}_{P_2+2,Q_2+2}\left[(qy)^{k_2}\left|\begin{matrix}(c_j,\kappa_j;C_j)_{1,N_2},(c_j,\kappa_j)_{N_2+1,P_2},(1-\beta,k_2),\,(d+e+f-\beta+1,k_2)\\(e-\beta+1,k_1),\,(f-\beta+1,k_2),\,(d_j,\tau_j)_{1,M_2},(d_j,\tau_j;D_j)_{M_2+1,Q_2}\end{matrix}\right.\right]$$

$$.\bar{H}^{M_3+2,N_3}_{P_3+2,Q_3+2}\left[(rz)^{k_3}\left|\begin{matrix}(e_j,\rho_j;E_j)_{1,N_3},(e_j,\rho_j)_{N_3+1,P_3},(1-\gamma,k_3),(g+h+k-\gamma+1,k_3)\\(h-\gamma+1,k_3),(k-\gamma+1,k_3),(f_j,\psi_j)_{1,M_3},(f_j,\psi_j;F_j)_{M_3+1,Q_3}\end{matrix}\right.\right]F(x,y,z)dxdydz$$

$$=\;\bar{H}^{M_1+2,N_1\;;\,M_2+2,N_2;\,M_3+2,N_3}_{P_1+2,Q_1+2\,;\,P_2+2,Q_2\,;\,P_3+2,Q_3}[\,F(x,y,z);\alpha,\beta,\gamma\,;\,p,q,r]$$

$$=\;\phi_1(p,q,r)$$

= R.H.S. of (4.2)
As far as the three dimensional Weyl type operators $J^{a,b,c}_{p,\infty}\,J^{d,e,f}_{q,\infty}\,J^{g,h,k}_{r,\infty}$ preserves the class u_3, it follows that ϕ_1 (p,q,r) also belongs to u_3.

It is interesting to note that the statement of Theorem 1 can easily be extended for arbitrary real a, d, g by using the definition (3.3) for the generalized Weyl type fractional calculus operators and differentiating under the signs of the integrals.

5. Interesting special cases

Taking c = f = k = 0 in Theorem 1, we have the following Theorem 1 (a).

Theorem 1(a). For $Re(a) > 0$, $Re(d) > 0$, $Re(g)$¿ 0, b ¿ 0, d ¿ 0, f ¿ 0, $k_1 > 0$, $k_2 > 0$, $k_3 > 0$ and also let ϕ (p,q,r) be given by (3.3) then the following formula

$$J^{a,b,0}_{p,\infty} J^{d,e,0}_{q,\infty} J^{g,h,0}_{r,\infty} [\phi(p,q,r) = \phi_2(p,q,r) \tag{5.1}$$

provided that ϕ_2 (p,q,r) exists and belongs to u_3 where ϕ_2 is represented by the repeated integral

$$\phi_2(p,q,r) = \int_{\lambda_1}^{\infty}\int_{\lambda_2}^{\infty}\int_{\lambda_3}^{\infty} (px)^{\alpha-1}(qy)^{\beta-1}(rz)^{\gamma-1}$$

$$.\bar{H}^{M_1+1,N_1}_{P_1+1,Q_1+1}\left[(px)^{k_1}\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,N_1},(a_j,\alpha_j)_{N_1+1,P_1},(a+b-\alpha+1,k_1)\\(b-\alpha+1,k_1),(b_j,\beta_j)_{1,M_1},(b_j,\beta_j)_{M_1+1,Q_1}\end{matrix}\right.\right]$$

$$.\bar{H}^{M_2+1,N_2}_{P_2+1,Q_2+1}\left[(qy)^{k_2}\left|\begin{matrix}(c_j,\kappa_j;C_j)_{1,N_2},(c_j,\kappa_j)_{N_2+1,P_2},(d+e-\beta+1,k_2)\\(e-\beta+1,k_2),(d_j,\tau_j)_{1,M_2},(d_j,\tau_j;D_j)_{M_2+1,Q_2}\end{matrix}\right.\right]$$

$$.\bar{H}^{M_3+1,N_3}_{P_3+1,Q_3+1}\left[(rz)^{k_3}\left|\begin{matrix}(e_j,\rho_j;E_j)_{1,N_3},(e_j,\rho_j)_{N_3+1,P_3},(g+h-\gamma+1,k_3)\\(h-\gamma+1,k_3),(f_j,\psi_j)_{1,M_3},(f_j,\psi_j;F_j)_{M_3+1,Q_3}\end{matrix}\right.\right] F(x,y,z)dxdydz \tag{5.2}$$

For $A_j = B_j = 1$, the $\bar{H}$-function in (3.5) reduces to Fox's H-function [5], [8] and then Theorem 1 (a) reduces to

$$J^{a,b,0}_{p,\infty} J^{d,e,0}_{q,\infty} J^{g,h,0}_{r,\infty} [\phi(p,q,r) = \phi_3(p,q,r) \tag{5.3}$$

provided that ϕ_3 (p,q,r) exists and belongs to u_3, where ϕ_3 is represented by the repeated integral

$$\phi_3(p,q,r) = \int_{\lambda_1}^{\infty}\int_{\lambda_2}^{\infty}\int_{\lambda_3}^{\infty} (px)^{\alpha-1}(qy)^{\beta-1}(rz)^{\gamma-1}$$

$$.\bar{H}^{M_1+1,N_1}_{P_1+1,Q_1+1}\left[(px)^{k_1}\left|\begin{matrix}(a_{P_1},\alpha_{P_1}),\ (a+b-\alpha+1,k_1)\\(b-\alpha+1,k_1),(b_{Q_1},\beta_{Q_1})\end{matrix}\right.\right]$$

$$.\bar{H}^{M_2+1,N_2}_{P_2+1,Q_2+1}\left[(qy)^{k_2}\left|\begin{matrix}(c_{P_2},\kappa_{P_2}),(d+e-\beta+1,k_2)\\(e-\beta+1,k_2),(d_{Q_2},\tau_{Q_2})\end{matrix}\right.\right]$$

$$.\bar{H}^{M_3+1,N_3}_{P_3+1,Q_3+1}\left[(rz)^{k_3}\left|\begin{matrix}(e_{P_3},\rho_{P_3}),(g+h-\gamma+1,k_3)\\(h-\gamma+1,k_3),(f_{Q_3},\psi_{Q_3})\end{matrix}\right.\right] F(x,y,z)dxdydz \tag{5.4}$$

On employing the identity

$$H^{M,N}_{P,Q}\left[x\left|\begin{matrix}(a_P,1)\\(b_Q,1)\end{matrix}\right.\right] = G^{M,N}_{P,Q}\left[x\left|\begin{matrix}a_1,\ldots,a_P\\b_1,\ldots,b_Q\end{matrix}\right.\right] \tag{5.5}$$

We see that the three dimensional H-transform reduces to the corresponding three dimensional G-transform ψ (p,q,r) defined by

$$\psi(p,q,r) = G^{M_1,N_1;M_2,N_2;M_3,N_3}_{P_1,Q_1;P_2,Q_2;P_3,Q_3}[F(x,y,z);\alpha,\ \beta,\ \gamma;\ p,q,r]$$

$$= \int_{\lambda_1}^{\infty} \int_{\lambda_2}^{\infty} \int_{\lambda_3}^{\infty} (px)^{\alpha-1} (qy)^{\beta-1} (rz)^{\gamma-1} G_{P_1,Q_1}^{M_1,N_1} \left[(px)^{k_1} \middle| \begin{smallmatrix} a_1,\ldots,a_{P_1} \\ b_1,\ldots,b_{Q_1} \end{smallmatrix} \right]$$

$$(5.6) \qquad .G_{P_2,Q_2}^{M_2,N_2} \left[(qy)^{k_2} \middle| \begin{smallmatrix} c_1,\ldots,c_{P_2} \\ d_1,\ldots,d_{Q_2} \end{smallmatrix} \right] G_{P_3,Q_3}^{M_3,N_3} \left[(rz)^{k_3} \middle| \begin{smallmatrix} e_1,\ldots,e_{P_3} \\ f_1,\ldots,f_{Q_3} \end{smallmatrix} \right] f(x,y,z)dxdydz$$

provided that ϕ_3 (p,q,r) exists and belongs to class u_3, where k_1, k_2 and k_3 are positive integers, $\lambda_1 > 0$, $\lambda_2 > 04$, $\lambda_3 > 0$, $P_1 \leqq Q_1$, $P_2 \leqq Q_2$, $P_3 \leqq Q_3$, $|argp^{k_1}| < \frac{T_1^*\pi}{2}$, $|\,argq^{k_2}| < \frac{T_2^*\pi}{2}$, $|argr^{k_3}| < \frac{T_3^*\pi}{2}\, withT_1^* = 2N_1+2M_1-P_1-Q_1$, $T_2^* = 2N_2+2M_2-P_2-Q_2$, $T_3^* = 2N_3+2M_3-P_3-Q_3\, G_{P,Q}^{M,N}[.]$ appealing in (5.6) and (5.7) represents Meijer's G-function whose detailed account is available from the monograph of Mathai and Saxena [7].

Thus, we obtain the following Theorem 1 (b).

Theorem 1 (b). For $Re(a) > 0$, $Re(d) > 0$, $Re(g)$¿ 0, b ¿ 0, d ¿ 0, f ¿ 0, k_1, k_2 and k_3 being positive integers and also let ψ (p,q,r) be given by (5.7) then the following formula

$$(5.7) \qquad J_{p,\infty}^{a,b,c} J_{q,\infty}^{d,e,f} J_{r,\infty}^{g,h,k} [\psi(p,q,r) = \psi_1(p,q,r)$$

holds, provided that ψ_1 (p,q,r) exists and belongs to class u_3 for other conditions on the parameters, in which additional parameters a, b, c, d, e, f, g, h, k included correspond to those in (5.7). Here

$$\psi_1(p,q,r) = k_1^{-a} k_2^{-d} k_3^{-g} \int_{\lambda_1}^{\infty} \int_{\lambda_2}^{\infty} \int_{\lambda_3}^{\infty} (px)^{\alpha-1} (qy)^{\beta-1} (rz)^{\gamma-1}$$

$$.G_{P_1+2,N_1}^{M_1+2,N_1} \left[(px)^{k_1} \middle| \begin{smallmatrix} a_1,\ldots,a_{P_1}, \Delta(k_1,1-\alpha),\, \Delta(k_1,a+b+c-\alpha+1) \\ \Delta(k_1,b-\alpha+1),\, \Delta(k_1,c-\alpha+1), b_1,\ldots,b_{Q_1} \end{smallmatrix} \right]$$

$$.G_{P_2+2,N_2}^{M_2+2,N_2} \left[(qy)^{k_2} \middle| \begin{smallmatrix} c_1,\ldots,c_{P_2}, \Delta(k_2,1-\beta),\, \Delta(k_2,d+e+f-\beta+1) \\ \Delta(k_2,e-\beta+1),\, \Delta(k_2,f-\beta+1), d_1,\ldots,d_{Q_2} \end{smallmatrix} \right]$$

$$(5.8) \qquad .G_{P_3+2,N_3}^{M_3+2,N_3} \left[(rz)^{k_3} \middle| \begin{smallmatrix} e_1,\ldots,e_{P_3}, \Delta(k_3,1-\gamma),\, \Delta(k_3,g+h+k-\gamma+1) \\ \Delta(k_3,h-\gamma+1),\, \Delta(k_3,k-\gamma+1), f_1,\ldots,f_{Q_3} \end{smallmatrix} \right] F(x,y,z)dxdydz$$

and the symbol Δ (n,α) represents the sequence of parameters

$$\frac{\alpha}{n},\ \frac{\alpha+1}{n}, \ldots, \frac{\alpha+n-1}{n}$$

On taking c = f = k = 0, (5.8) becomes

$$(5.9) \qquad J_{p,\infty}^{a,b,0} J_{q,\infty}^{d,e,0} J_{r,\infty}^{g,h,0} [\psi(p,q,r) = \psi_2(p,q,r)$$

provided φ_3 (p,q,r) exists and belongs to class u_3, where φ_3 is represented by the integral

$$\varphi_3(p,q,r) = k_1^{-a} k_2^{-d} k_3^{-g} \int_{\lambda_1}^{\infty} \int_{\lambda_2}^{\infty} \int_{\lambda_3}^{\infty} (px)^{\alpha-1} (qy)^{\beta-1} (rz)^{\gamma-1}$$

$$.G^{M_1+1,N_1}_{P_1+1,N_1}\left[(px)^{k_1}\left|{}^{a_1,\ldots,a_{P_1},\,\Delta(k_1,a+b-\alpha+1)}_{\Delta(k_1,b-\alpha+1),\,b_1,\ldots,b_{Q_1}}\right.\right]$$

$$.G^{M_2+1,N_2}_{P_2+1,N_2}\left[(qy)^{k_2}\left|{}^{c_1,\ldots,c_{P_2},\,\Delta(k_2,d+e-\beta+1)}_{\Delta(k_2,e-\beta+1),\,d_1,\ldots,d_{Q_2}}\right.\right]$$

$$.G^{M_3+1,N_3}_{P_3+1,N_3}\left[(rz)^{k_3}\left|{}^{e_1,\ldots,e_{P_3},\,\Delta(k_3,g+h-\gamma+1)}_{\Delta(k_3,h-\gamma+1),\,f_1,\ldots,f_{Q_3}}\right.\right]F(x,y,z)dxdydz \tag{5.10}$$

6. Two-Dimensional Analogue of Theorem 1

The following two dimensional analogue can be established on the similar lines as given in Theorem 1.

Theorem 2. Let ϕ (p,q) be the two-dimensional $\bar{H}$ transform of F (x,y) defined by

$$\phi\,(p,q)=\phi^{M_1,N_1;\,M_2,N_2}_{P_1,Q_1;\,P_2,Q_2}[F(x,y);\alpha,\ \beta\,;p,q]$$

$$=\int_{\lambda_1}^{\infty}\int_{\lambda_2}^{\infty}(px)^{\alpha-1}\,(qy)^{\beta-1}\,\bar{H}^{M_1,N_1}_{P_1,Q_1}\left[(px)^{k_1}\left|{}^{(a_j,\alpha_j;A_j)_{1,N_1},(a_j,\alpha_j)_{N_1+1,P_1}}_{(b_j,\beta_j)_{1,M_1},(b_j,\beta_j;B_j)_{M_1+1,Q_1}}\right.\right]$$

$$.\,\bar{H}^{M_2+2,N_2}_{P_2+2,Q_2+2}\left[(qy)^{k_2}\left|{}^{(c_j,\kappa_j;C_j)_{1,N_2},(c_j,\kappa_j)_{N_2+1,P_2}}_{(d_j,\tau_j)_{1,M_2},(d_j,\tau_j;D_j)_{M_2+1,Q_2}}\right.\right]F(x,y)dxdy \tag{6.1}$$

provided that ϕ (p,q) exists and belongs to u_2, where $k_1>0$, $k_2>0$, $|argp^{k_1}|<\frac{1}{2}T_1\pi$, $|argq^{k_2}|<\frac{1}{2}T_2\pi$ and

$$F(x,y)=f\left[a\sqrt{x^2-\lambda_1^2}\,H(x-\lambda_1),\ c\sqrt{y^2-\lambda_2^2}\,H(y-\lambda_2)\right]\ x>\lambda_1>0,y>\lambda_2>0$$

and H (t) denotes Heaviside's unit step function.

Then for $Re(a)>0$, $Re(d)>0$, $\lambda_1>0$, $\lambda 2$ ¿ 0, $k_1>0$, $k_2>0$ and let ϕ_1 (p,q) be defined as

$$\phi_1\,(p,q)=\int_{\lambda_1}^{\infty}\int_{\lambda_2}^{\infty}(px)^{\alpha-1}\,(qy)^{\beta-1}$$

$$.\,\bar{H}^{M_1+2,N_1}_{P_1+2,Q_1+2}\left[(px)^{k_1}\left|{}^{(a_j,\alpha_j;A_j)_{1,N_1},\,(a_j,\alpha_j)_{N_1+1,P_1},\,(1-\alpha,k_1),\,(a+b+c-\alpha+1,k_1)}_{(b-\alpha+1,k_1),\ (c-\alpha+1,k_1),(b_j,\beta_j)_{1,M_1},(b_j,\beta_j;\,B_j)_{M_1+1,Q_1}}\right.\right]$$

$$.\,\bar{H}^{M_2+2,N_2}_{P_2+2,Q_2+2}\left[(qy)^{k_2}\left|{}^{(c_j,\kappa_j;C_j)_{1,M_2},(c_j,\kappa_j)_{N_2+1,P_2},\,(1-\beta,k_2),(d+e+f-\beta+1,k_2)}_{(e-\beta+1,k_2),(f-\beta+1,k_2),\ (d_j,\tau_j)_{1,M_2},(d_j,\tau_j;D_j)_{M_2+1,Q_2}}\right.\right]F(x,y)dxdy$$

then the following formula

$$\bar{J}^{a,b,c}_{p,\infty}\,\bar{J}^{d,e,f}_{q,\infty}\,[\phi\,(p,q)]=\phi_1\,(p,q) \tag{6.2}$$

holds, provided that ϕ_1(p,q) exists and belongs to class u_2.

Here $\bar{J}^{a,b,c}_{p,\infty}\,\bar{J}^{d,e,f}_{q,\infty}\,[f(p,q)]=\frac{p^b\,q^e}{\Gamma(a)\Gamma(d)}\int_p^{\infty}\int_q^{\infty}(u-p)^{a-1}(v-q)^{d-1}u^{-a-b}v^{-d-e}$

$$. \, {}_2F_1\left(a+b, -c; a; 1-\frac{p}{u}\right) \, {}_2F_1\left(d\varepsilon, -f; d; 1-\frac{q}{v}\right) \, f(u,v)dudv$$

7. Special Case

For $A_j = B_j = 1$, the $\bar{H}$-function in (3.5) reduces to Fox's H-function and then (6.2) gives us the result obtained by Saigo, Saxena and Ram [14, p.67].

8. One dimensional Analogue of Theorem 1

Similar proof with Theorem 1 can be developed for the following one-dimensional analogue

Theorem 3. Let ϕ (p) be the one dimensional $\bar{H}$-transform of F (x) defined by

$$\phi\,(p) = \phi_{P,Q}^{M,N}\,[F(x);\alpha\,;\,p]$$

$$= \int_{\lambda_1}^{\infty} (px)^{\alpha-1}\, \bar{H}_{P,Q}^{M,N}\left[(px)^{k_1}\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,N},\,(a_j,\alpha_j)_{N+1,P}\\(b_j,\beta_j)_{1,M},\,(b_j,\beta_j;B_j)_{M+1,Q}\end{matrix}\right.\right]\, F(x)dx \tag{8.1}$$

provided that ϕ (p) exists and belongs to class u_1, where $k_1 > 0$, $|arg\beta^{k_1}| < \frac{1}{2}T\pi$,

$$F(x) = f(a\sqrt{x^2-\lambda_1^2}\,)\,H(x-\lambda_1) \tag{8.2}$$

For $Re(a) > 0$, $\lambda_1 > 0$, and let ϕ_1 (p) be defined as

$$\phi_1\,(p) = \int_{\lambda_1}^{\infty} (px)^{\alpha-1}\, \bar{H}_{P+2,Q+2}^{M+2,N}\left[\begin{matrix}(a_j,\alpha_j;A_j)_{1,N},(a_j,\alpha_j)_{N+1,P},\,(1-\alpha,k_1),(a+b+c-\alpha+1,k_1)\\(b-\alpha+1,k_1),\,(c-\alpha+1,k_1),(b_j,\beta_j)_{1,M},(b_j,\beta_j;B_j)_{M+1,Q}\end{matrix}\right] F(x)dx$$

Then the following formula

$$\bar{J}_{p,\infty}^{a,bc}[\phi\,(p)] = \phi_1\,(p) \tag{8.3}$$

holds, provided that ϕ_1 (p) exists and belongs to class u_1

Here $\bar{J}_{p,\infty}^{a,b,c}[f(p)] = \frac{p^b}{\Gamma(a)}\int_p^{\infty}(u-p)^{a-1}\,u^{-a-b}\,{}_2F_1\left(a+b, -c; a; 1-\frac{p}{u}\right) f(x)dx$

$$= \; p^b\, J_{p,\infty}^{a,bc}[f(p)].$$

Special Case

For $A_j = B_j = 1$, the $\bar{H}$-function in (3.5) reduces to Fox's H-function and then (8.3) reduces to the result obtained by Saigo, Saxena and Ram [14, p.70]. On account of the most general character of the $\bar{H}$-function, numerous interesting special cases of the results established in this paper can be obtained by suitably specializing the parameters of the$\bar{H}$-function.

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*Department of Mathematics,
University of Rajasthan,
Jaipur - 302055, India
E-mail address: `chaurasiavbl@yahoo.in`

**Department of Mathematics,
Yagyavalkya Institute of Technology
Jaipur-302022, India.
E-mail address: `Monika Jain monikaj@gmail.com`