SCIENTIA
Series A: *Mathematical Sciences*, Vol. 19 (2010), 1–6
Universidad Técnica Federico Santa María
Valparaíso, Chile
ISSN 0716-8446

Certain integral properties pertaining to special functions

V.B.L. Chaurasia[a] and Jagdev Singh [b]

Abstract. The aim of the present paper is to study certain integral properties of M-series [7], a general class of polynomials [9], Fox's H-function [3] and $\bar{H}$-function [6] which contains a certain class of Feynman integrals, exact partition of a Gaussian model in statistical mechanics and several other functions as its particular cases. The main results of our paper are unified in nature and capable of yielding a very large number of corresponding interesting results (new and known) involving simpler special functions and polynomials as special cases of our formulae. The results established here are basic in nature and are likely to find useful application in several fields notably electrical networks, probability theory and statistical mechanics etc.

1. Introduction

The conventional formulation may fail pertaining to the domain of quantum cosmology but Feynman path integrals apply [5 and 6]. Feynman path integrals reformulation of quantum mechanics is more fundamental than the conventional formulation in terms of operators. Feynman integrals are useful in the study and development of simple and multiple variable hypergeometric series which in turn are useful in statistical mechanics.

The M-series is defined by Sharma [7] as

$$\begin{aligned} {}_s\overset{\alpha}{M}_t(u_1, ..., u_s; v_1, ..., v_t; w) &= {}_s\overset{\alpha}{M}_t(w) \\ &= \sum_{k=0}^{\infty} \frac{(u_1)_k ... (u_s)_k}{(v_1)_k ... (v_t)_k} \frac{w^k}{\Gamma(\alpha k + 1)} \end{aligned} \tag{1.1}$$

here $\alpha \in C, \mathrm{Re}(\alpha) > 0,$ and $(u_j)_k, (v_j)_k$ are the Poacheammar symbols. The series (1.1) is defined when none of the parameters v_js, j = l, 2,...,q is negative integer or zero. If any numerator parameter α_j is a negative or zero, the series terminates to

2000 *Mathematics Subject Classification.* 33C60, 26A33.

Key words and phrases. Feynman integrals, Laguerre polynomials, Hermite polynomials, Jacobi polynomials, Fox's H-function, $\bar{H}$-function, M-Series.

The authors are highly thankful to Professor H.M. Srivastava, University of Victoria, Victoria, Canada, for his kind help and many valuable suggestions in the preparation of this present paper.

a polynomial in w. From the ratio test it is evident that the series (1.1) is convergent for all w if $s \leqslant t$, if it convergent for $s = t+1$ and divergent,if $s > t+1$. When $s = t+1$ and $|w| = 1$, the series can converge in some cases. Let $\beta = \sum_{j=1}^{s} u_j - \sum_{j=1}^{t} v_j$. Let $\beta = \sum_{j=1}^{s} u_j - \sum_{j=1}^{t} v_j$.It can be shown that when $s = t+1$ the series is absolutely convergent of $|w| = 1$ if $Re(\beta) < 0$, conditionally convergent for $w = -1$ if $0 \leqslant Re(\beta) < 1$ and divergent for $|w| = 1$ if $1 \leqslant Re(\beta)$.

2. Main Results

Here we shall derive the following results involving a general class of polynomials [9], for H-function [3] (Skibinski [8]), M-series [7] with $\bar{H}$-function [6].

$$\begin{aligned}
&\int_0^1\int_0^1 \left(\tfrac{1-x}{1-xy}y\right)^{\sigma}\left(\tfrac{1-y}{1-xy}\right)^{\rho}\tfrac{1-xy}{(1-x)(1-y)}S_{n'}^{m'}\left[a_1\left(\tfrac{1-x}{1-xy}y\right)^{\alpha_1}\left(\tfrac{1-y}{1-xy}\right)^{\beta_1}\right]\\
&\times H_{P,Q}^{M,N}\left[a_2\left(\tfrac{1-x}{1-xy}y\right)^{\alpha_2}\left(\tfrac{1-y}{1-xy}\right)^{\beta_2}\left|\begin{smallmatrix}(e_p,E_P)\\(f_Q,F_Q)\end{smallmatrix}\right.\right]\\
&\times\ {}_s\overset{\alpha}{M}_t\left[a_3\left(\tfrac{1-x}{1-xy}\right)^{\alpha_3}\left(\tfrac{1-y}{1-xy}\right)^{\beta_3}\right]\bar{H}_{p,q}^{m,n}\left[a_4\left(\tfrac{1-x}{1-xy}y\right)^{\alpha_4}\left(\tfrac{1-y}{1-xy}\right)^{\beta_4}\right]dxdy
\end{aligned}$$

$$\begin{aligned}
&=\sum_{r=0}^{[n'/m']}\sum_{g=1}^{M}\sum_{G,k=0}^{\infty}\tfrac{(-n')_{m'r}}{r!}A_{n',r}\tfrac{(-1)^G\varphi(\eta_G)}{G!F_g}a_1^r a_2^{\eta_G}\tfrac{(u_1)_k\ldots(u_s)_k}{(v_1)_k\ldots(v_t)_k}\tfrac{a_3^k}{\Gamma(\alpha k+1)}\\
&\times\bar{H}_{p+2,q+1}^{m,n+2}\left[a_4\left|\begin{smallmatrix}(1-\sigma-\alpha_1r-\alpha_2\eta_G-\alpha_3k,\alpha_4;1),(1-\rho-\beta_1r-\beta_2\eta_G-\beta_3k,\beta_4;1),(\alpha_j,A_j;a_j)_{1,n}(\alpha_j,A_j)_{n+1,p}\\(\beta_j,B_j)_{1,m}(\beta_j,B_j;b_j)_{m+1,q},[1-\sigma-\rho-(\alpha_1+\beta_1)r-(\alpha_2+\beta_2)\eta_G-(\alpha_3+\beta_3)k,\alpha_4+\beta_4;1\end{smallmatrix}\right.\right]
\end{aligned}\quad(2.1)$$

Provided that

$$\begin{aligned}
&\operatorname{Re}\left(\sigma+1+\alpha_2\tfrac{f_j}{F_j}+\alpha_4\tfrac{\beta_{j'}}{B_{j'}}\right)>0,\operatorname{Re}\left(\rho+1+\beta_2\tfrac{f_j}{F_j}+\beta_4\tfrac{\beta_{j'}}{B_{j'}}\right)>0,\\
&|\arg a_2|<\tfrac{1}{2}T\pi,T>0,|\arg a_1|<\tfrac{1}{2}T'\pi,T'>0,
\end{aligned}$$

where $j = 1,..,M; j' = 1,....,m; (\rho,\sigma,\alpha_1,\beta_1,\alpha_2,\beta_2,\alpha_3,\beta_3,\alpha_4,\beta_4) > 0$ *and* $s \leqslant t, |a_3| < 1, a_1 > 0$.

$$\begin{aligned}
&\int_0^\infty\int_0^\infty\varphi(u+v)u^{\sigma-1}v^{\rho-1}S_{n'}^{m'}[a_1u^{\alpha_1}v^{\beta_1}]H_{P,Q}^{M,N}\left[a_2u^{\alpha_2}v^{\beta_2}\left|\begin{smallmatrix}(e_p,E_p)\\(f_Q,F_Q)\end{smallmatrix}\right.\right]\\
&\times\ {}_s\overset{\alpha}{M}_t\left[a_3u^{\alpha_3}v^{\beta_3}\right]\bar{H}_{p,q}^{m,n}\left[a_4u^{\alpha_4}v^{\beta_4}\right]du\,dv\\
&=\sum_{r=0}^{[n'/m']}\sum_{g=1}^{M}\sum_{G,k=0}^{\infty}\tfrac{(-n')_{m'r}}{r!}A_{n',r}\tfrac{(-1)^G\varphi(\eta_G)}{G!F_g}\tfrac{(u_1)_k\ldots.(u_s)_k}{(v_1)_k\ldots.(v_t)_k}\tfrac{1}{\Gamma(\alpha k+1)}
\end{aligned}$$

$$\begin{aligned}
&.a_1^r a_2^{\eta_G}a_3^k\int_0^\infty\varphi(z)z^{\sigma+\rho+(\alpha_1+\beta_1)r+(\alpha_2+\beta_2)\eta_G+(\alpha_3+\beta_3)k-1}dz\\
&\times\bar{H}_{p+2,q+1}^{m,n+2}\left[a_4z^{\alpha_4+\beta_4}\left|\begin{smallmatrix}(1-\sigma-\alpha_1r-\alpha_2\eta_G-\alpha_3k,\alpha_4;1),(1-\rho-\beta_1r-\beta_2\eta_G-\beta_3k,\beta_4;1),(\alpha_j,A_j;a_j)_{1,n}(\alpha_j,A_j)_{n+1,p}\\(\beta_j,B_j)_{1,m}(\beta_j,B_j;b_j)_{m+1,q},[1-\sigma-\rho-(\alpha_1+\beta_1)r-(\alpha_2+\beta_2)\eta_G-(\alpha_3+\beta_3)k,\alpha_4+\beta_4;1\end{smallmatrix}\right.\right]
\end{aligned}\quad(2.2)$$

Provided that

$$\mathrm{Re}\left(\sigma+\alpha_2\frac{f_j}{F_j}+\alpha_4\frac{\beta_{j'}}{B_{j'}}\right)>0, \mathrm{Re}\left(\sigma+\alpha_2\left(\frac{e_j-1}{E_j}\right)+\alpha_4\left(\frac{a_{j'}(\alpha_{j'}-1)}{A_{j'}}\right)\right)<0,$$
$$\mathrm{Re}\left(\rho+\beta_2\frac{f_j}{F_j}+\beta_4\frac{\beta_{j'}}{B_{j'}}\right)>0, \mathrm{Re}\left(\rho+\beta_2\left(\frac{e_j-1}{E_j}\right)+\beta_4\left(\frac{a_{j'}(\alpha_{j'}-1)}{A_{j'}}\right)\right)<0$$
$$|\arg a_2|<\tfrac{1}{2}T\pi, T>0|\arg a_4|<\tfrac{1}{2}T'\pi, T'>0,$$

where $j=1,...,M; j'=1,...,m; (\rho,\sigma,\alpha_1,\beta_1,\alpha_2,\beta_2,\alpha_3,\beta_3,\alpha_4,\beta_4)>0$ and $s\leqslant t, |a_3|<1, a_1>0$.

$$\int_0^1\int_0^1 f(uv)(1-u)^{\sigma-1}(1-v)^{\rho-1}v^{\sigma}\ S_{n'}^{m'}[a_1(1-u)^{\alpha_1}(1-v)^{\beta_1}v^{\alpha_1}]$$
$$\times H_{P,Q}^{M,N}\left[a_2(1-u)^{\alpha_2}(1-v)^{\beta_2}v^{\alpha_2}\left|\begin{matrix}(e_p,E_p)\\(f_Q,F_Q)\end{matrix}\right.\right] {}_s\overset{\alpha}{M}_t[a_3(1-u)^{\alpha_3}(1-v)^{\beta_3}v^{\alpha_3}]$$
$$\times\ \bar{H}_{p,q}^{m,n}\left[a_4(1-u)^{\alpha_4}(1-v)^{\beta_4}v^{\alpha_4}\right]dx\,dy$$
$$=\sum_{r=0}^{[n'/m']}\sum_{g=1}^{M}\sum_{G,k=0}^{\infty}\frac{(-n')_{m'r}}{r!}A_{n',r}\frac{(-1)^G\varphi(\eta_G)}{G!F_g}\frac{(u_1)_k...(u_s)_k}{(v_1)_k...(v_t)_k}\frac{1}{\Gamma(\alpha k+1)}$$
$$.a_1^r a_2^{\eta_G}a_3^k\int_0^1 f(z)(1-z)^{\sigma+\rho+(\alpha_1+\beta_1)r+(\alpha_2+\beta_2)\eta_G+(\alpha_3+\beta_3)k-1}dz$$
$$\times\bar{H}_{p+2,q+1}^{m,n+2}\left[a_4(1-z)^{\alpha_4+\beta_4}\left|\begin{matrix}(1-\sigma-\alpha_1r-\alpha_2\eta_G-\alpha_3k,\alpha_4;1),(1-\rho-\beta_1r-\beta_2\eta_G-\beta_3k,\beta_4;1),(\alpha_j,A_j;a_j)_{1,n}(\alpha_j,A_j)_{n+1,p}\\(\beta_j,B_j)_{1,m}(\beta_j,B_j;b_j)_{m+1,q},[1-\sigma-\rho-(\alpha_1+\beta_1)r-(\alpha_2+\beta_2)\eta_G-(\alpha_3+\beta_3)k,\alpha_4+\beta_4;1\end{matrix}\right.\right] \tag{2.3}$$

Provided that

$$\mathrm{Re}\left(\sigma+1+\alpha_2\frac{f_j}{F_j}+\alpha_4\frac{\beta_{j'}}{B_{j'}}\right)>0, \mathrm{Re}\left(\alpha+\alpha_2\frac{f_j}{F_j}+\alpha_4\frac{\beta_{j'}}{B_{j'}}\right)>0,$$
$$\mathrm{Re}\left(\rho+\beta_2\frac{f_j}{F_j}+\beta_4\frac{\beta_{j'}}{B_{j'}}\right)>0, |\arg a_2|<\tfrac{1}{2}T\pi, T>0, |\arg a_4|<\tfrac{1}{2}T'\pi, T'>0,$$

where $j=1,...,M; j'=1,...,m; (\rho,\sigma,\alpha_1,\beta_1,\alpha_2,\beta_2,\alpha_3,\beta_3,\alpha_4,\beta_4)>0$ *and* $s\leqslant t, |a_3|<1, a_1>0$.

$$\int_0^1\int_0^1\left[\frac{y(1-x)}{(1-xy)}\right]^{\alpha+\sigma}\left[\frac{1-y}{1-xy}\right]^{\beta}\frac{1}{(1-x)}P_n^{(\alpha,\beta)}\left[1-2a_1\left(\frac{y(1-x)}{(1-xy)}\right)^{\alpha_1}\left(\frac{1-y}{1-xy}\right)^{\beta_1}\right]$$
$$.\,S_{n'}^{m'}\left[a_2\left(\frac{y(1-x)}{(1-xy)}\right)^{\alpha_2}\left(\frac{1-y}{1-xy}\right)^{\beta_2}\right]H_{P,Q}^{M,N}\left[a_3\left(\frac{y(1-x)}{(1-xy)}\right)^{\alpha_3}\left(\frac{1-y}{1-xy}\right)^{\beta_3}\left|\begin{matrix}(e_p,E_P)\\(f_Q,F_Q)\end{matrix}\right.\right]$$
$$.\ {}_s\overset{\alpha'}{M}_t\left[a_4\left(\frac{y(1-x)}{(1-xy)}\right)^{\alpha_4}\left(\frac{1-y}{1-xy}\right)^{\beta_4}\right]\bar{H}_{p,q}^{m,n}\left[a_5\left(\frac{y(1-x)}{(1-xy)}\right)^{\alpha_5}\left(\frac{1-y}{1-xy}\right)^{\beta_5}\right]dxdy$$
$$=\sum_{r=0}^{[n'/m']}\sum_{g=1}^{M}\sum_{G,k=0}^{\infty}\frac{(-n')_{m'r}}{r!}A_{n',r}\frac{(-1)^G\varphi(\eta_G)}{G!F_g}\frac{(u_1)_k...(u_s)_k}{(v_1)_k...(v_t)_k}\frac{1}{\Gamma(\alpha'k+1)}$$
$$.a_2^r a_3^{\eta_G}a_4^k\frac{(1+\alpha)_n}{n!}\ {}_2F_1[-n,\alpha+\beta+n+1;1+\alpha;a_1]$$
$$\times\bar{H}_{p+2,q+1}^{m,n+2}\left[a_5\left|\begin{matrix}(1-\alpha-\sigma-\alpha_1k_1-\alpha_2r-\alpha_3\eta_G-\alpha_4k,\alpha_5;1),(-\beta-\beta_1k_1-\beta_2r-\beta_3\eta_G-\beta_4k,\beta_5;1),(\alpha_j,A_j;a_j)_{1,n}(\alpha_j,A_j)_{n+1,p}\\(\beta_j,B_j)_{1,m}(\beta_j,B_j;b_j)_{m+1,q},[-\alpha-\beta-\sigma-(\alpha_1+\beta_1)k_1-(\alpha_2+\beta_2)r-(\alpha_3+\beta_3)\eta_G-(\alpha_4+\beta_4)k,\alpha_5+\beta_5;1\end{matrix}\right.\right], \tag{2.4}$$

Provided that

$$\mathrm{Re}\left(\alpha+\sigma+1+\alpha_3\frac{f_j}{F_j}+\alpha_5\frac{\beta_{j'}}{B_{j'}}\right)>0, \mathrm{Re}\left(\beta+1+\beta_3\frac{f_j}{F_j}+\beta_5\frac{\beta_{j'}}{B_{j'}}\right)>0,$$
$$|\arg a_3|<\tfrac{1}{2}T\pi, T>0, |\arg a_5|<\tfrac{1}{2}T'\pi, T'>0,$$

where

$j = 1, ..., M; j' = 1, ..., m; (\alpha, \beta, \sigma, \alpha_1, \beta_1, \alpha_2, \beta_2, \alpha_3, \beta_3, \alpha_4, \beta_4, \alpha_5, \beta_5) > 0$ *and* $s \leqslant t$, $s \leqslant t, |a_4| < 1$.

Proofs. The results (2.1) through (2.4) can be easily established by using the same technique as used by Chaurasia and Shekhawat [2].

3. Particular cases

(A) For the generalized hypergeometric function [7] when $\alpha = 1$, we have ${}_s\overset{1}{M}_t(w) = \sum_{k=0}^{\infty} \frac{(u_1)_k.....(u_s)_k}{(v_1)_k....(v_t)_k} \frac{w^k}{k!} = {}_sF_t(w)$ the results (2.1) through (2.4) reduce to the following formulae

$$\text{(A.1)} \quad \int_0^1\int_0^1 \left(\frac{1-x}{1-xy}y\right)^{\sigma} \left(\frac{1-y}{1-xy}\right)^{\rho} \frac{(1-xy)}{(1-x)(1-y)} S_{n'}^{m'}\left[a_1\left(\frac{1-x}{1-xy}y\right)^{\alpha_1}\left(\frac{1-y}{1-xy}\right)^{\beta_1}\right]$$
$$\times H_{P,Q}^{M,N}\left[a_2\left(\frac{1-x}{1-xy}y\right)^{\alpha_2}\left(\frac{1-y}{1-xy}\right)^{\beta_2}\middle|\begin{matrix}(e_p,E_P)\\(f_Q,F_Q)\end{matrix}\right]$$
$$\times \ {}_sF_t\left[a_3\left(\frac{1-x}{1-xy}y\right)^{\alpha_3}\left(\frac{1-y}{1-xy}\right)^{\beta_3}\right]\bar{H}_{p,q}^{m,n}\left[a_4\left(\frac{1-x}{1-xy}y\right)^{\alpha_4}\left(\frac{1-y}{1-xy}\right)^{\beta_4}\right]dxdy$$

$$= \sum_{r=0}^{[n'/m']}\sum_{g=1}^{M}\sum_{G,k=0}^{\infty} \frac{(-n')_{m'r}}{r!}A_{n',r}\frac{(-1)^G\varphi(\eta_G)}{G!F_g}\frac{(u_1)_k....(u_s)_k}{(v_1)_k....(v_t)_k}a_1^r a_2^{\eta_G}a_3^k\frac{1}{k!}$$
$$\bar{H}_{p+2,q+1}^{m,n+2}\left[a_4\middle|\begin{matrix}(1-\sigma-\alpha_1 r-\alpha_2\eta_G-\alpha_3 k,\alpha_4;1),(1-\rho-\beta_1 r-\beta_2\eta_G-\beta_3 k,\beta_4;1),(\alpha_j,A_j;a_j)_{1,n}(\alpha_j,A_j)_{n+1,p}\\(\beta_j,B_j)_{1,m}(\beta_j,B_j;b_j)_{m+1,q},[1-\sigma-\rho-(\alpha_1+\beta_1)r-(\alpha_2+\beta_2)\eta_G-(\alpha_3+\beta_3)k,\alpha_4+\beta_4;1]\end{matrix}\right]$$

valid under the same condition as needed for (2.1).

$$\text{(A.2)} \quad \int_0^{\infty}\int_0^{\infty}\varphi(u+v)u^{\sigma-1}v^{\rho-1}\ S_{n'}^{m'}[a_1u^{\alpha_1}v^{\beta_1}]H_{P.Q}^{M,N}\left[a_2u^{\alpha_2}v^{\beta_2}\middle|\begin{matrix}(e_p,E_p)\\(f_Q,F_Q)\end{matrix}\right]$$
$$.{}_sF_t\left[a_3u^{\alpha_3}v^{\beta_3}\right]\bar{H}_{p,q}^{m,n}\left[a_4u^{\alpha_4}v^{\beta_4}\right]du\,dv$$

$$= \sum_{r=0}^{[n'/m']}\sum_{g=1}^{M}\sum_{G,k=0}^{\infty} \frac{(-n')_{m'r}}{r!}A_{n',r}\frac{(-1)^G\varphi(\eta_G)}{G!F_g}\frac{(u_1)_k....(u_s)_k}{(v_1)_k....(v_t)_k}.\frac{1}{k!}$$
$$.a_1^r a_2^{\eta_G}a_3^k\int_0^{\infty}\varphi(z)z^{\sigma+\rho+(\alpha_1+\beta_1)r+(\alpha_2+\beta_2)\eta_G+(\alpha_3+\beta_3)k-1}dz$$
$$\times\bar{H}_{p+2,q+1}^{m,n+2}\left[a_4z^{\alpha_4+\beta_4}\middle|\begin{matrix}(1-\sigma-\alpha_1 r-\alpha_2\eta_G-\alpha_3 k,\alpha_4;1),(1-\rho-\beta_1 r-\beta_2\eta_G-\beta_3 k,\beta_4;1),(\alpha_j,A_j;a_j)_{1,n}(\alpha_j,A_j)_{n+1,p}\\(\beta_j,B_j)_{1,m}(\beta_j,B_j;b_j)_{m+1,q},[1-\sigma-\rho-(\alpha_1+\beta_1)r-(\alpha_2+\beta_2)\eta_G-(\alpha_3+\beta_3)k,\alpha_4+\beta_4;1\end{matrix}\right],$$

exists under the same conditions as required for (2.2).

$$(\mathrm{A}.3)\qquad \int_0^1\int_0^1 f(uv)(1-u)^{\sigma-1}(1-v)^{\rho-1}v^{\sigma}S_{n'}^{m'}[a_1(1-u)^{\alpha_1}(1-v)^{\beta_1}v^{\alpha_1}]$$
$$\times H_{P,Q}^{M,N}\left[a_2(1-u)^{\alpha_2}(1-v)^{\beta_2}v^{\alpha_2}\left|\begin{matrix}(e_p,E_P)\\(f_Q,F_Q)\end{matrix}\right.\right]{}_sF_t[a_3(1-u)^{\alpha_3}(1-v)^{\beta_3}v^{\alpha_3}]$$
$$\times\ \bar{H}_{p,q}^{m,n}\left[a_4(1-u)^{\alpha_4}(1-v)^{\beta_4}v^{\alpha_4}\right]dx\,dy$$
$$=\sum_{r=0}^{[n'/m']}\sum_{g=1}^{M}\sum_{G,k=0}^{\infty}\frac{(-n')_{m'r}}{r!}A_{n',r}\frac{(-1)^G\varphi(\eta_G)}{G!F_g}\frac{(u_1)_k\ldots(u_s)_k}{(v_1)_k\ldots(v_t)_k}\frac{1}{k!}$$
$$.a_1^r a_2^{\eta_G}a_3^k\int_0^1 f(z)(1-z)^{\sigma+\rho+(\alpha_1+\beta_1)r+(\alpha_2+\beta_2)\eta_G+(\alpha_3+\beta_3)k-1}dz$$
$$\times\bar{H}_{p+2,q+1}^{m,n+2}\left[a_4(1-z)^{\alpha_4+\beta_4}\left|\begin{matrix}(1-\sigma-\alpha_1r-\alpha_2\eta_G-\alpha_3k,\alpha_4;1),(1-\rho-\beta_1r-\beta_2\eta_G-\beta_3k,\beta_4:1),(\alpha_j,A_j;a_j)_{1,n}(\alpha_j,A_j)_{n+1,p}\\(\beta_j,B_j)_{1,m}(\beta_j,B_j;b_j)_{m+1,q},[1-\sigma-\rho-(\alpha_1+\beta_1)r-(\alpha_2+\beta_2)\eta_G-(\alpha_3+\beta_3)k,\alpha_4+\beta_4;1]\end{matrix}\right.\right],$$

valid under the same conditions as obtainable from (2.3).

$$(\mathrm{A}.4)\qquad \int_0^1\int_0^1\left[\frac{y(1-x)}{(1-xy)}\right]^{\alpha+\sigma}\left[\frac{1-y}{1-xy}\right]^{\beta}\frac{1}{(1-x)}P_n^{(\alpha,\beta)}\left[1-2a_1\left(\frac{y(1-x)}{(1-xy)}\right)^{\alpha_1}\left(\frac{1-y}{1-xy}\right)^{\beta_1}\right]$$
$$.\,S_{n'}^{m'}\left[a_2\left(\frac{y(1-x)}{(1-xy)}\right)^{\alpha_2}\left(\frac{1-y}{1-xy}\right)^{\beta_2}\right]H_{P,Q}^{M,N}\left[a_3\left(\frac{y(1-x)}{(1-xy)}\right)^{\alpha_3}\left(\frac{1-y}{1-xy}\right)^{\beta_3}\left|\begin{matrix}(e_p,E_P)\\(f_Q,F_Q)\end{matrix}\right.\right]$$
$$.\ {}_sF_t\left[a_4\left(\frac{y(1-x)}{(1-xy)}\right)^{\alpha_4}\left(\frac{1-y}{1-xy}\right)^{\beta_4}\right]\bar{H}_{p,q}^{m,n}\left[a_5\left(\frac{y(1-x)}{(1-xy)}\right)^{\alpha_5}\left(\frac{1-y}{1-xy}\right)^{\beta_5}\right]dx\ dy$$
$$=\sum_{r=0}^{[n'/m']}\sum_{g=1}^{M}\sum_{G,k=0}^{\infty}\frac{(-n')_{m'r}}{r!}A_{n',r}\frac{(-1)^G\varphi(\eta_G)}{G!F_g}\frac{(u_1)_k\ldots.(u_s)_k}{(v_1)_k\ldots.(v_t)_k}\frac{1}{k!}$$
$$.a_2^r a_3^{\eta_G}a_4^k\frac{(1+\alpha)_n}{n!}\,{}_2F_1[-n,\alpha+\beta+n+1;1+\alpha;a_1]$$
$$\times\bar{H}_{p+2,q+1}^{m,n+2}\left[a_5\left|\begin{matrix}(1-\sigma-\alpha_1k_1-\alpha_2r-\alpha_3\eta_G-\alpha_4k,\alpha_5;1),(-\beta-\beta_1k_1-\beta_2r-\beta_3\eta_G-\beta_4k,\beta_5;1),(\alpha_j,A_j;a_j)_{1,n}(\alpha_j,A_j)_{n+1,p}\\(\beta_j,B_j)_{1,m}(\beta_j,B_j;b_j)_{m+1,q},[-\alpha-\beta-\sigma-(\alpha_1+\beta_1)k_1-(\alpha_2+\beta_2)r-(\alpha_3+\beta_3)\eta_G-(\alpha_4+\beta_4)k,\alpha_5+\beta_5;1]\end{matrix}\right.\right]$$

valid under the same conditions as surrounding (2.4).

(B) Taking $\alpha_1\to 1,\beta_1\to 0,\alpha_2\to 0,\beta_2\to 0,a_2\to 1,\alpha_3\to 0,\beta_3\to 0,\alpha_4\to 1$ in equation (2.1) $a_1\to 0,\alpha_1\to 1,\beta_1\to 0,a_2\to 1,\alpha_2\to 0,\beta_2\to 0,\ \alpha_3\to 0,$ in equation (2.2) and (2.3), we get the known results recently obtained by Chaurasia and Shekhawat ([2], eqn. (2.1), p.2, eqn. (2.3), p.3 and eqn. (2.5), p.4).

References

[1] Buschman, R.G.; Srivastava, H.M. - The $\bar{H}$-function associated with a certain class of Feynman integrals, J. Phys. A: Math. Gen. 23 (1990), 4707-4710.

[2] Chaurasia, V.B.L.; Shekhawat, A.S. - Some integral properties of a general class of polynomials associated with Feynman integrals, Bull. Malays. Math. Sci. Soc. (2), 28(2) (2005), 1-7.

[3] Fox, C. The G and H-functions as symmetrical Fourier kernels, Trans. Amer. Math. Soc. 98 (1961), 395-429.

[4] Grosche, C.; Steiner, F. - Hand book of Feynman path integrals, Springer tracts in modern physics, Vol.145, Springer-Verlag, Berlin Heidelberg, New York (1913).

[5] Inayat-Hussain, A.A. - New properties of hypergeometric series derivable from Feynman integrals: 1. Transformations and reduction formulae, J. Phys. A : Math. Gen. 20 (1987), 4109-4117

[6] Inayat-Hussain, A.A. - New properties of hypergeometric derivable from Feynman integrals : II. A generalization of the H-function, J. Phys. A : Math. 20 (1987), 4119-4128.

[7] Sharma, M. - Fractional integration and fractional differentiation of the M-series, Frac. Cal. Appl. Anal., 11 (2008), 187-191.
[8] Skibinski, P. - Some expansion theorems for the H-functions, Ann. Polon. Math., 23 (1970), 1-6.
[9] Srivastava, H.M. - A contour integral involving Fox H-function, Indian J. Math. 14 (1972), 1-6.
[10] SrivastavaA, H.M.; Singh, N.P. - The integration of certain products of the multivariable H-function with a general class of polynomials, Rend. Circ. Mat. Palermo 2(32) (1983), 157-187.
[11] Szego, C. - Orthogonal polynomials, Amer. Math. Soc. Colloq. Publ. 23, Fourth edition, Amer. Math. Soc. Providence, Rhode Island (1975).

Received 02 06 2009, revised 12 04 2010

[a]Department of Mathematics,
University of Rajasthan, Jaipur - 302055,
Rajasthan, India
E-mail address: `chaurasiavbl@yahoo.in`

[b]Department of Mathematics,
Jagan Nath University, Village Rampura,
Tehsil Chaksu, Jaipur - 303901,
Rajasthan, India
E-mail address: `jagdevsinghrathore@yahoo.com`