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A NOTE ON SEQUENCE-COVERING IMAGES OF METRIC SPACES

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Abstract. In this brief note, we prove that every space is a sequence-covering image of a topological sum of convergent sequences. As the application of this result, sequence-covering images of locally compact metric spaces (or, locally separable metric spaces, metric spaces) and sequentially-quotient images of a locally compact metric spaces (or, locally separable metric spaces, metric spaces) are equivalent.

In [6], Z. Li and Y. Ge proved the following theorem.

Theorem 1 ([6], Theorem 6)**.** *Let X be a space. Then there exists a metric space M and a pseudo-sequence-covering mapping $f : M \longrightarrow X$.*

By using this result, the authors obtained that pseudo-sequence-covering images of metric spaces and sequentially-quotient images of metric spaces are equivalent. After that, Y. Ge [4] showed the equivalence of sequence-covering images and sequentially-quotient images for metric domains as follows.

Theorem 2 ([4], Theorem 7)**.** *The following are equivalent for a space X.*

(1) *X is a sequence-covering image of a metric space.*
(2) *X is a pseudo-sequence-covering image of a metric space.*
(3) *X is a sequentially-quotient image of a metric space.*

Take these results into account, note that "sequence-covering $\Longrightarrow$ pseudo-sequence-covering", and "locally compact metric $\Longrightarrow$ locally separable metric $\Longrightarrow$ metric", then the following questions are natural.

Question 3. *Can "pseudo-sequence-covering" in Theorem 1 be replaced by "sequence-covering"?*

Question 4. *Can "metric" in Theorem 2 be replaced by "locally separable metric" or "locally compact metric"?*

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In this brief note, we answer Question 3 affirmatively by proving that every space is a sequence-covering image of a topological sum of convergent sequences. As the application of this result, sequence-covering images of locally compact metric spaces (or, locally separable metric spaces, metric spaces) and sequentially-quotient images of a locally compact metric spaces (or, locally separable metric spaces, metric spaces) are equivalent, which answers Question 4 affirmatively.

Throughout this paper, all spaces are assumed to be Hausdorff and all mappings are continuous and onto, $\mathbb{N}$ denotes the set of all natural numbers, ω denote $\mathbb{N} \cup \{0\}$, and a convergent sequence includes its limit point.

Let $f : X \longrightarrow Y$ be a mapping.

f is a *sequence-covering mapping* [8], if every convergent sequence of Y is the image of some convergent sequence of X.

f is a *pseudo-sequence-covering mapping* [5], if every convergent sequence of Y is the image of some compact subset of X.

f is a *subsequence-covering mapping* [7], if for every convergent sequence S of Y, there is a compact subset K of X such that $f(K)$ is a subsequence of S.

f is a *sequentially-quotient mapping* [1], if for every convergent sequence S of Y, there is a convergent sequence L of X such that $f(L)$ is a subsequence of S.

For terms which are not defined here, please refer to [2].

Theorem 5. *Let X be a space. Then there exists a topological sum M of convergent sequences and a sequence-covering mapping $f : M \longrightarrow X$.*

Proof. Let $S(X)$ be the collection of all convergent sequences in X. For each $S \in S(X)$, put $S = \{x_n : n \in \omega\}$ with the limit point x_0 and $S(x_0) = \{(x_n, S) : n \in \omega\}$. Then $S(x_0)$ is a convergent sequence in $X \times S(X)$ and all $S(x_0)$'s are distinct. Put $M = \bigoplus_{S \in S(X)} S(x_0)$ and define $f : M \longrightarrow X$ by choosing $f((x_n, S)) = x_n$ for every $n \in \omega$. Then M is a topological sum of convergent sequences and f is a sequence-covering mapping from M onto X. □

Corollary 6. *The following are equivalent for a space X.*

(1) *X is a sequence-covering image of a locally compact metric space.*
(2) *X is a pseudo-sequence-covering image of a locally compact metric space.*
(3) *X is a subsequence-covering image of a locally compact metric space.*
(4) *X is a subsequentially-quotient image of a locally compact metric space.*
(5) *X is a sequence-covering image of a locally separable metric space.*
(6) *X is a pseudo-sequence-covering image of a locally separable metric space.*
(7) *X is a subsequence-covering image of a locally separable metric space.*
(8) *X is a sequentially-quotient image of a locally separable metric space.*
(9) *X is a sequence-covering image of a metric space.*
(10) *X is a pseudo-sequence-covering image of a metric space.*
(11) *X is a subsequence-covering image of a metric space.*
(12) *X is a sequentially-quotient image of a metric space.*

Proof. (1) $\Longrightarrow$ (2) $\Longrightarrow$ (3) $\Longrightarrow$ (4), (5) $\Longrightarrow$ (6) $\Longrightarrow$ (7) $\Longrightarrow$ (8), (9) $\Longrightarrow$ (10) $\Longrightarrow$ (11) $\Longrightarrow$ (12), (1) $\Longrightarrow$ (5)$\Longrightarrow$ (9), (4) $\Longrightarrow$ (8) $\Longrightarrow$ (12). By definitions of mappings and [3, Lemma 10].

(12) $\Longrightarrow$ (1). By Theorem 5, note that each topological sum of convergent sequences is a locally compact metric space. □

Remark 7. Theorem 5 and Corollary 6 are generalizations of [6, Theorem 6] and [4, Theorem 7], respectively.

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