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BEM-FEM coupling for wave-structure interaction

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Abstract. We present a numerical method, based on a coupling of finite elements and boundary elements, to solve a fluid–solid interaction problem posed in the plane. The boundary unknowns involved in our formulation are approximated by a spectral method. We provide error estimates for the Galerkin method and present numerical results that illustrate the accuracy of our scheme.

1. Introduction

We introduce a numerical method based on the coupling of finite elements (FEM) and boundary elements (BEM) to describe the interaction between a bounded solid body and the compressible inviscid fluid surrounding it, when time–harmonic excitations of the system are imposed.

We follow [**1, 3, 7**] and use linear integral equations as nonlocal boundary conditions on an artificial interface. In [**1, 3**] the boundary that separates the two media (the wet interface) is used as a coupling boundary. The well posedness of the resulting formulation (at the continuous level) requires regularity assumptions for the wet interface that may not be fulfilled in practice. In the present paper, we proceed as in [**7**] and impose the absorbing boundary conditions on a smooth but arbitrary interface that contains the obstacle in its interior. This procedure enlarges the domain for finite element computations, however, this drawback is compensated by the fact that our scheme is no more limited to problems with smooth wet boundaries.

We also point out that the coupling methods proposed in [**1, 3, 7**] lead to formulations that are not well-posed when the square of the wave number is a Dirichlet eigenvalue of the Laplace operator in the interior domain. Thus, the corresponding numerical method may exhibit an unstable behavior in the vicinity of these singular coefficients. The method presented here is free from this restriction.

The paper is organized as follows. In sections 2 and 3, we introduce the model problem, derive a coupled boundary–interior formulation and prove its well-posedness.

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In section 5 we introduce the Galerkin BEM-FEM discretization and provide a convergence result. Finally, section 5 is devoted to the numerical results.

1.1. Notations and Sobolev spaces. We will extensively deal with complex valued functions. The symbol $\imath$ is used for $\sqrt{-1}$. For a complex number $\alpha \in \mathbb{C}$, $\overline{\alpha}$ denotes its conjugate and $|\alpha|$ its modulus. If Ω is bounded open set in $\mathbb{R}^2$, $\|\cdot\|_{0,\Omega}$ denotes the $L^2(\Omega)$-norm. More generally, for any $m \in \mathbb{N}$, $\|\cdot\|_{m,\Omega}$ denotes the usual norm of the Sobolev space $H^m(\Omega)$, see [**2**].

We will also consider periodic Sobolev spaces. Let $C^\infty_{2\pi}$ be the space of 2π-periodic and infinitely differentiable complex valued functions of a single variable. Given $g \in C^\infty_{2\pi}$, we define its Fourier coefficients

$$\widehat{g}(k) := \frac{1}{2\pi}\int_0^{2\pi} g(s)e^{-\imath k s}\,ds. \tag{1.1}$$

Then, for $p \in \mathbb{R}$, we define the Sobolev space H^p to be the completion of $C^\infty_{2\pi}$ with respect to the norm $\|g\|_p := \left(\sum_{k\in\mathbb{Z}}(1+|k|^2)^p|\widehat{g}(k)|^2\right)^{1/2}$. It is well known (see [**4**]) that H^p are Hilbert spaces and $H^p \subset H^q$ for every $p > q$, the inclusion being dense and compact. Notice that $H^0 = L^2(0,\,2\pi)$. The H^0-bilinear form

$$\langle\, \lambda, \eta \,\rangle := \int_0^{2\pi} \lambda(t)\,\eta(t)\,dt$$

can be extended to represent the duality between H^{-p} and H^p for all $p > 0$. We will keep the same notation for this duality bracket.

2. Governing equations

Consider an elastic body, modelled as an infinitely long cylinder (parallel to the x_3-axis) whose cross section is Ω_s. The boundary of Ω_s is denoted Σ and we assume that the exterior of the body is occupied by a fluid. We are concerned with the response of the fluid–solid system to the action of time–harmonic forces on the solid and of a time–harmonic wave travelling in the fluid. The variables of the problem are the spacial components of the displacement field for the solid and the scattered wave in the fluid.

Let ω be the frequency of the incident wave and of the body forces and let the amplitudes of those be denoted respectively by $\mathrm{w} = \mathrm{w}(x_1, x_2)$ and $\mathbf{f} = \mathbf{f}(x_1, x_2)$. The incident wave is generally taken to satisfy the Helmholtz equation $\Delta\mathrm{w} + k^2\mathrm{w} = 0$ in $\Omega_f := \mathbb{R}^2 \setminus \overline{\Omega_s}$.

We assume that the phenomenon is invariant under a translation in the x_3-direction and consider a bidimensional model posed in the frequency domain. The unknowns of the problem are the amplitude $\mathbf{u} : \Omega_s \to \mathbb{C}^2$ of the solid displacements field and the amplitude $p : \Omega_f \to \mathbb{C}$ of the scattered pressure.

We suppose that the solid is homogeneous, isotropic and linearly elastic, with mass density ρ_s and Lamé constants λ, μ. We denote as usual the stress tensor by $\sigma(\mathbf{u}) := \lambda\,\mathrm{tr}\varepsilon(\mathbf{u})\,I + 2\,\mu\,\varepsilon(\mathbf{u})$, where $\varepsilon_{ij}(\mathbf{u}) := \frac{1}{2}(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i})$ is the infinitesimal strain tensor. Furthermore, we assume that the fluid is ideal, compressible and homogeneous with mass density ρ_f and wave number $k = \frac{\omega}{c}$ where c is the speed of sound in the linearized fluid.

Let us denote by $\mathbf{n}$ the unit normal on Σ directed into Ω_f. Under the hypothesis of small oscillations both in the solid and the fluid, $\mathbf{u}$ and p are found out to satisfy the equations

$$\left\{\begin{array}{rcll} \nabla\cdot\sigma(\mathbf{u}) + \rho_s\,\omega^2\,\mathbf{u} &=& -\mathbf{f} & \text{in } \Omega_s, \\ \Delta p + k^2\,p &=& 0 & \text{in } \Omega_f, \\ \sigma(\mathbf{u})\mathbf{n} &=& -(p+\mathrm{w})\,\mathbf{n} & \text{on } \Sigma, \\ \rho_f\,\omega^2\,\mathbf{u}\cdot\mathbf{n} &=& \frac{\partial(p+\mathrm{w})}{\partial\mathbf{n}} & \text{on } \Sigma, \end{array}\right. \tag{2.1}$$

and the decay condition

$$\frac{\partial p}{\partial r} - \imath k p = o(r^{-1/2}) \tag{2.2}$$

when $r := |\boldsymbol{x}| \to +\infty$ uniformly for all directions $\frac{\boldsymbol{x}}{|\boldsymbol{x}|}$.

It is known that if $\mathbf{f} = \mathbf{0}$ and $\mathrm{w} = 0$ then $p = 0$ and $\mathbf{u}$ is solution of (see [**5**])

$$\left\{\begin{array}{rcll} \nabla\cdot\sigma(\mathbf{u}) + \rho_s\,\omega^2\,\mathbf{u} &=& 0 & \text{in } \Omega_s, \\ \sigma(\mathbf{u})\mathbf{n} &=& 0 & \text{on } \Sigma, \\ \mathbf{u}\cdot\mathbf{n} &=& 0 & \text{on } \Sigma. \end{array}\right. \tag{2.3}$$

It turns out that for certain regions and some frequencies $\rho_s\,\omega^2$, known as *Jones frequencies*, problem (2.3) has nontrivial solutions. This seems to be a rare eventuality but we will, in any case, assume that (2.3) only admits the trivial solution.

3. A week formulation with non-local boundary conditions

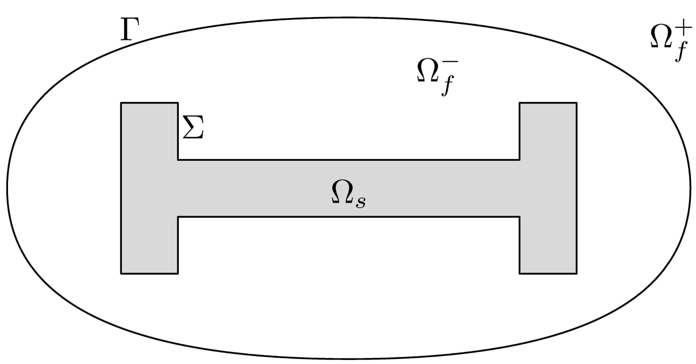

FIGURE 1. Geometry of the problem.

Let us introduce an artificial boundary Γ such that Ω_s lies in its interior. Then, Γ separates $\mathbb{R}^2$ into a bounded domain Ω and an unbounded region Ω_f^+ exterior to Γ. We denote $\Omega_f^- := \Omega_f \cap \Omega$. Notice that $\overline{\Omega} = \overline{\Omega_s} \cup \overline{\Omega_f^-}$. The normal vector on Γ is always taken pointing into Ω_f^+, see Figure 1.

We consider the bilinear forms

$$E^\omega(\mathbf{u},\mathbf{v}) := \int_{\Omega_s} (\sigma(\mathbf{u}) : \varepsilon(\mathbf{v}) \,-\, \rho_s \omega^2 \mathbf{u}\cdot\mathbf{v})\, d\boldsymbol{x},$$

$$a^k(p,q) := \int_{\Omega_f^-} (\nabla p \cdot \nabla q \,-\, k^2\, p\, q)\, d\boldsymbol{x} \qquad \text{and} \qquad D(\mathbf{v},q) := \int_\Sigma \mathbf{v}\cdot\mathbf{n}\, q\, d\tau.$$

We are concerned here with the following BEM-FEM formulation of (2.1)–(2.2) introduced in [**10**]:

$$\text{(3.1)} \qquad \left\{ \begin{array}{rcll} \multicolumn{4}{l}{\text{find } \mathbf{u} \in (H^1(\Omega_s))^2,\ p \in H^1(\Omega_f^-),\ \xi \in H^{-1/2} \text{ and } \psi \in H^{1/2} \text{ such that}} \\ E^\omega(\mathbf{u},\mathbf{v}) + D(\mathbf{v},p) &=& L(\mathbf{v}) & \forall \mathbf{v} \in (H^1(\Omega_s))^2 \\ a^k(p,q) + \rho_f \omega^2 D(\mathbf{u},q) - \langle\, \xi, \gamma q \,\rangle &=& \ell(q) & \forall q \in H^1(\Omega_f^-) \\ \langle\, \mu, \gamma p \,\rangle - \frac{1}{2}\langle\, \mu, \psi \,\rangle - b(\psi,\mu) - \imath\eta s(\psi,\mu) &=& 0 & \forall \mu \in H^{-1/2} \\ \langle\, \xi, \varphi \,\rangle + c(\psi,\varphi) + \imath\frac{\eta}{2}\langle\, \psi,\varphi \,\rangle - \imath\eta b(\varphi,\psi) &=& 0 & \forall \varphi \in H^{1/2} \end{array} \right.$$

where $\eta > 0$ is arbitrary,

$$L(\mathbf{v}) := \int_{\Omega_s} \mathbf{f}\cdot\mathbf{v}\, d\boldsymbol{x} - D(\mathbf{v},\mathrm{w}), \qquad \ell(q) := \int_\Sigma \frac{\partial \mathrm{w}}{\partial \mathbf{n}}\, q\, d\tau,$$

$$b(\psi,\mu) := \langle\, \mu, \mathcal{D}\psi \,\rangle, \qquad c(\psi,\varphi) := \langle\, \mathcal{H}\psi,\varphi \,\rangle \quad \text{and} \quad s(\psi,\mu) := \langle\, \mu, \mathcal{S}\psi \,\rangle.$$

We used here the single and double layer acoustic potentials

$$\mathcal{S}g(s) := \int_0^{2\pi} V(s,t) g(t) dt \qquad \text{and} \qquad \mathcal{D}g(s) := \int_0^{2\pi} K(s,t) g(t) dt,$$

where

$$V(s,t) := \frac{\imath}{4} H_0^{(1)}(k|\boldsymbol{x}(s) - \boldsymbol{x}(t)|)$$

and

$$K(s,t) := -\frac{k\imath}{4} H_1^{(1)}(k|\boldsymbol{x}(t) - \boldsymbol{x}(s)|) \frac{x_2'(t)(x_1(t) - x_1(s)) - x_1'(t)(x_2(t) - x_2(s))}{|\boldsymbol{x}(t) - \boldsymbol{x}(s)|},$$

$H_0^{(1)}$ and $H_1^{(1)}$ being the Hankel functions of the first kind and order zero and one respectively.

Finally, we define the hypersingular operator $\mathcal{H}$ through the following identity that relate it to the single layer operator: (see [**6**, page 295])

$$\text{(3.2)} \qquad \langle\, \mathcal{H}\psi, \eta \,\rangle = \langle\, \eta', \mathcal{S}\psi' \,\rangle - k^2 \langle\, \widetilde{\mathcal{S}}\psi, \eta \,\rangle \quad \forall \psi, \eta \in H^{1/2},$$

where $\widetilde{\mathcal{S}}$ is the integral operator whose kernel is given by $\widetilde{V}(t,s) := \mathbf{x}'(t)\cdot\mathbf{x}'(s) V(t,s)$.

We point out that the auxiliary unknown ξ is given in terms of the normal derivative of p on Γ by $\xi := |\boldsymbol{x}'|\,\frac{\partial p}{\partial \boldsymbol{\nu}} \circ \boldsymbol{x}$ while ψ is a density function that is not directly related to p.

Theorem 1. *Problem* (3.1) *is well posed.*

Proof. See [**10**]. ☐

4. The discrete scheme

For simplicity of exposition, from now on we assume that Σ is polygonal. Let N be a given integer. We consider the equidistant subdivision $\{t_i := i\pi/N; \quad i = 0, \cdots, 2N-1\}$ of the interval $[0,\, 2\pi]$ with $2N$ grid points. We denote by Ω_h the polygonal domain whose vertices lying on Γ are $\{\mathbf{x}(t_i) :\ i = 0, \ldots, 2N-1\}$. Let $\{\tau_h\}$ be a regular family of triangulations of $\overline{\Omega}_h$ by triangles T of diameter h_T not greater than $\max ||\mathbf{x}'(\cdot)||\, h$, where $h := \pi/N$. We assume that the restriction $\tau_h^s := \left\{T \in \tau_h; \quad T \subset \overline{\Omega_s}\right\}$ of τ_h to $\overline{\Omega_s}$ is a triangulation, i.e., that τ_h respects the interface between Ω_s and Ω_f^-. If we set $\tau_h^f := \tau_h \setminus \tau_h^s$, then $\Omega_{f,h}^- :=$ interior$(\cup_{T\in\tau_h^f} T)$ is a polygonal approximation of Ω_f^-.

From τ_h^f we can obtain a triangulation $\widetilde{\tau}_h^f$ of $\overline{\Omega_f^-}$ by replacing each triangle of τ_h^f with one side along $\partial\Omega_h$ by the corresponding curved triangle. Let then T be a curved triangle in $\widetilde{\tau}_h^f$. It is well known that there exists $h_0 > 0$ such that if $h \in (0, h_0)$, T is the range of $\widehat{T}$ by a C^∞ and one-to-one mapping $\mathbf{F}_T : \widehat{T} \to \mathbb{R}^2$ that may be computed explicitly in terms of $\boldsymbol{x}$. This type of diffeomorfism was first proposed by Zlámal [**11**]. If T is a straight (interior) triangle, $\mathbf{F}_T$ is the usual affine map from the reference triangle. This hypothesis will be implicit in the following.

Let $(\widehat{T}, P_m(\widehat{T}), \widehat{\Sigma}_m)$ denote the standard Lagrange finite element of order m on the reference triangle $\widehat{T}$. A finite element is defined on T by a triplet $(T, P_m(T), \Sigma_m)$, where $P_m(T)$ is the image under $\mathbf{F}_T$ of the space $P_m(\widehat{T})$ of polynomials of degree no greater than m on $\widehat{T}$:

$$P_m(T) := \{p :\ T \to \mathbb{C};\ p = \hat{p} \circ \mathbf{F}_T^{-1},\ \hat{p} \in P_m(\widehat{T})\},$$

and $\Sigma_T = \{N_i^k; \quad i = 1, \cdots, (m+1)(m+2)/2\}$ is a set of linear functionals defined by $N_i(\phi) = \phi \circ \mathbf{F}_T(\widehat{\mathbf{a}}_i) \quad \forall \phi \in \mathcal{C}^0(T)$ where $\widehat{\mathbf{a}}_i$ are the nodes in $\widehat{T}$.

We introduce the finite element spaces

$$V_h^s := \{v \in \mathcal{C}^0(\overline{\Omega_s}); \quad v|_T \in P_m(T)\ \ \forall T \in \tau_h^s\}$$

and

$$V_h^f := \{q \in \mathcal{C}^0(\overline{\Omega_f^-}); \quad q|_T \in P_m(T)\ \ \forall T \in \widetilde{\tau}_h^f\},$$

Finally, for any integer n, we consider the $2n$–dimensional space

$$T_n := \left\{ \sum_{j=0}^{n} a_j \cos jt + \sum_{j=1}^{n-1} b_j \sin jt; \quad a_j, b_j \in \mathbb{C} \right\}.$$

The discrete version of (3.1) is given by

$$\text{(4.1)}\quad \begin{cases} \text{find } \mathbf{u}_h \in (V_h^s)^2,\ p_h \in V_h^f,\ \xi_n \in T_n \text{ and } \psi_n \in T_n \text{ such that} \\ \begin{array}{rcll} E^\omega(\mathbf{u}_h, \mathbf{v}) + D(\mathbf{v}, p_h) & = & L(\mathbf{v}) & \forall \mathbf{v} \in (V_h^s)^2 \\ a^k(p_h, q) + \rho_f \omega^2 D(\mathbf{u}_h, q) - \langle\, \xi_n, \gamma q \,\rangle & = & \ell(q) & \forall q \in V_h^f \\ \langle\, \mu, \gamma p_h \,\rangle - \frac{1}{2}\langle\, \mu, \psi_n \,\rangle - b(\psi_n, \mu) - \imath\eta s(\psi_n, \mu) & = & 0 & \forall \mu \in T_n \\ \langle\, \xi_n, \varphi \,\rangle + c(\psi_n, \varphi) + \imath\frac{\eta}{2}\langle\, \psi_n, \varphi \,\rangle - \imath\eta b(\varphi, \psi_n) & = & 0 & \forall \varphi \in T_n \end{array} \end{cases}$$

Theorem 2. *If $\delta := (h, \frac{1}{n})$ is small enough, problem (4.1) has a unique solution. Moreover, if $\mathbf{u} \in H^{m+1}(\Omega_s)^2$ and $p \in H^{m+1}(\Omega_f^-)$, then*

$$\|\widehat{\mathbf{u}} - \widehat{\mathbf{u}}_\delta\|_{\mathbf{M}} \leqslant C_2\big(h^m(\|\mathbf{u}\|_{m+1,\Omega_s} + \|p\|_{m+1,\Omega_f^-}) + (2/n)^\sigma(\|\xi\|_{\sigma-1/2} + \|\psi\|_{\sigma+1/2})\big) \quad (\forall \sigma > 0),$$

where $\widehat{\mathbf{u}} := (\mathbf{u}, p, \xi, \psi)$, $\widehat{\mathbf{u}}_\delta := (\mathbf{u}_h, p_h, \xi_n, \psi_n)$ and $\mathbf{M} := H^1(\Omega_s)^2 \times H^1(\Omega_f^-) \times H^{-1/2} \times H^{1/2}$.

Proof. See [**10**]. □

5. Numerical results

We test our numerical method with a problem (2.1) whose exact solution is known explicitly. We take $\Omega_s = (-0.2, 0.2) \times (-0.4, 0.4)$ and define Γ to be the ellipse centered at the origin with minor and major semiaxis equal to 0.4 and 0.6 respectively. We also choose $\rho_s = \rho_f = c = \lambda = \mu = 1$. In sequel, $k^2 = (\omega/c)^2 = 25.9948$ is an approximation of the first Dirichlet eigenvalue of the Laplace operator in Ω. Thus, our method is tested in the case where the numerical schemes given in [**1, 3, 7**] may exhibit an unstable behavior because of the lack of uniqueness. Let us denote by K_0, K_1 and K_2 the modified Bessel functions of the second kind and order 0, 1 and 2 respectively. The function given by

$$\mathbf{u}_e(\boldsymbol{x}) = \frac{1}{2\pi} \begin{pmatrix} \psi(\boldsymbol{x}) - \frac{(x_1-0.3)^2}{r_1^2}\chi(\boldsymbol{x}) \\ -\frac{(x_1-0.3)x_2}{r_1^2}\chi(\boldsymbol{x}) \end{pmatrix}, \qquad \left(r_1 := \sqrt{(x_1-0.3)^2 + x_2^2}\right),$$

with $\psi(\boldsymbol{x}) := K_0(\imath\omega r_1) + \frac{1}{\imath\omega r_1}\left(K_1(\imath\omega r_1) - \frac{1}{\sqrt{3}}K_1(\frac{\imath\omega r_1}{\sqrt{3}})\right)$ and $\chi(\boldsymbol{x}) := K_2(\imath\omega r_1) - \frac{1}{3}K_2(\frac{\imath\omega r_1}{\sqrt{3}})$, is a solution of the elastodynamic equation in Ω_s when $\mathbf{f} = \mathbf{0}$.

On the other hand, the scalar function $p_e(\boldsymbol{x}) = H_0^{(1)}(\omega|\boldsymbol{x}|)$ solves the Helmholtz equation in Ω_f and satisfies the radiation conditions (2.2). Thus, $(\mathbf{u}_e, p_e)$ is solution of (2.1) with non-homogeneous transmission conditions on Σ.

In Table 1, we take $h = 2\pi/64$ and $\omega = \sqrt{25.9948}$ and we use a quadratic finite element method, i.e., we take $m = 2$. We decrease the spectral parameter n until we obtain the smallest value that preserves the order of accuracy. We can see that the number of degrees of freedom on the boundary may be drastically reduced without affecting the convergence of the scheme. This justifies the following strategy used to solve the linear systems of equations: We use a static condensation method to eliminate

$2n$	$\|\mathbf{u}-\mathbf{u}_h^*\|_{1,\Omega_s}$	$\|p-p_h^*\|_{1,\Omega_f^-}$
64	6.66×10^{-4}	1.99×10^{-3}
48	6.68×10^{-4}	1.93×10^{-3}
32	7.28×10^{-4}	6.44×10^{-3}
24	5.54×10^{-3}	1.19×10^{-2}

Table 1. Convergence history and number of iterations of the method for different values of the parameter n when $h=2\pi/64$ and $\omega=\sqrt{25.9948}$.

h	$\|\mathbf{u}-\mathbf{u}_h^*\|_{1,\Omega_s}$	$\|p-p_h^*\|_{1,\Omega_s}$	iterations
$2\pi/24$	4.82×10^{-3}	1.46×10^{-2}	22
$2\pi/32$	3.14×10^{-3}	6.83×10^{-3}	22
$2\pi/48$	1.29×10^{-3}	3.15×10^{-3}	22
$2\pi/64$	6.77×10^{-4}	2.07×10^{-3}	22

Table 2. Convergence history and number of iterations of the method for different values of the parameter h when $\omega=\sqrt{25.9948}$ and $n=20$.

the boundary variables. Afterwards, the reduced system is solved by a preconditioned GMRES method with the preconditioner

$$\begin{pmatrix} \mathbf{A}_0 & \mathbf{0} \\ \mathbf{0} & \mathbf{R}_0 \end{pmatrix}$$

where $\mathbf{A}_0$ and $\mathbf{R}_0$ are the matrices associated to the sesquilinear forms $\int_{\Omega_s}\sigma(\mathbf{u}):\varepsilon(\mathbf{v})\,d\boldsymbol{x}$ and $\int_{\Omega_f^-}\nabla p\cdot\nabla q\,d\boldsymbol{x}$ respectively. We use a version of GMRES without restarts. We take as an initial guess an identically vanishing function in both Ω_s and Ω_f^-. Iterations are continued until $\|r_{k+1}\|_2/\|r_k\|_2<10^{-6}$ where r_k is the k–th residual. Table 2 shows the number of iterations against h with $n=20$ and $\omega=5$. The numerical results suggest that the method has a number of iterations bounded independently of the critical parameter h.

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