SCIENTIA
Series A: *Mathematical Sciences*, Vol. 26 (2015), 141–153
Universidad Técnica Federico Santa María
Valparaíso, Chile
ISSN 0716-8446

ON I-ALMOST CONTINUITY AND NEW DECOMPOSITION OF CONTINUITY

XIAOMIN LI

Abstract. In this paper, I-almost continuous functions are introduced and their characterizations and properties are given. Relationships between I-almost continuity and other forms of continuity are discussed. Moreover, new decomposition of continuity with I-almost continuity as a factor is obtained.

1. Introduction

General topology as a branch of mathematics is concerned with all questions directly or indirectly related to continuity. The study of continuity have been found to be useful in many fields of applications such as computer science and digital topology [**?**, **?**]. Therefore, generalization of continuity is one of the most important subjects in general topology.

Ideal topological spaces were studied by Kuratowski [**?**] and Vaidyanathaswamy [**?**]. The purpose of this paper is to study continuity in ideal topological spaces. We introduce the concept of I-almost continuous functions and give their characterizations and properties. We also discuss the relationships between I-almost continuity and other forms of continuity. In addition, we obtain a new decomposition of continuity with I-almost continuity as a factor.

2. Preliminaries

Let X be a set and 2^X denotes the power set of X. A subset I of 2^X is called an ideal on X if I satisfies the follow conditions:

(1) $A \in I$ and $B \subset A$ implies $B \in I$;
(2) $A \in I$ and $B \in I$ implies $A \cup B \in I$.

If I is an ideal on X and τ is a topology on X, then a pair (X, τ, I) is called an ideal topological space or simply an ideal space.

2010 *Mathematics Subject Classification.* Primary 54C05, 54C08.
Key words and phrases. Ideal spaces; I-almost continuity; R-I-open sets; Almost continuity; Weakly I-continuity; Decompositions of continuity.
This work is supported by the National Natural Science Foundation of China (11461005).

Let (X, τ, I) be an ideal space. A operator $(.)^* : 2^X \longrightarrow 2^X$, called a local function [?] of A with respect to τ and I, is defined as follows: for any $A \subset X$,

$$A^*(I, \tau) = \{x \in X : V \cap A \notin I \ for\ every\ V \in \tau(x)\},$$

where $\tau(x) = \{U \in \tau : x \in U\}$.

A operator $cl^*(.) : 2^X \longrightarrow 2^X$ is defined as follows: for any $A \subset X$,

$$cl^*(A)(I, \tau) == A \cup A^*(I, \tau).$$

Because $cl^*(.)$ is a Kuratowski closure operator, thus $cl(.)^*$ generates a topology $\tau^*(I, \tau)$, called $*$-topology.

It is easy to prove that $\tau^*(I, \tau) \supset \tau$.

When there is no chance for confusion, we will simply write τ^* for $\tau^*(I, \tau)$, A^* for $A^*(I, \tau)$, c^*A for $cl^*(A)(I, \tau)$ and i^*A for $int^*(A)(I, \tau)$, where $int^*(A)(I, \tau) = X - cl^*(X - A)(I, \tau)$.

A is called $*$-closed [?] if $c^*A = A$, and A is called $*$-open (i.e., $A \in \tau^*$) if $X - A$ is $*$-closed. Obviously, A is $*$-open if and only if $i^*A = A$.

Throughout this paper, spaces always mean topological spaces or ideal spaces on which no separation axiom is assumed. Sometimes, (X, τ) and (X, τ, I) are simply written by X. If $\mathcal{U} \subset 2^X$, $A \subset X$ and $x \in X$, then $\mathcal{U}_A$ denotes $\{U \bigcap A : U \in \mathcal{U}\}$ and $\mathcal{U}(x)$ denotes $\{U \in \mathcal{U} : x \in U\}$. If A is a subset of a space X, then the closure of A and the interior of A denote by cA and iA respectively, and we have

$$iA \subset i^*A \subset A \subset c^*A \subset cA.$$

Let $f_i : 2^X \longrightarrow 2^X$ be a operator $(i = 1, 2, \cdots, n)$ and $A \subset X$. We define

$$f_1 f_2 \cdots f_n A = f_1(f_2(\cdots(f_n(A))\cdots)).$$

Definition 2.1. *A subset A of a space (X, τ) is called*

(1) *regular open* [?] *if $A = icA$ and regular closed if $X - A$ is regular open.*

(2) *pre-open* [?] *if $A \subset icA$ and pre-closed if $X - A$ is pre-open.*

(3) *semi-open* [?] *if $A \subset ciA$ and semi-closed if $X - A$ is semi-open.*

Definition 2.2. *A subset A of a space (X, τ, I) is called*

(1) *R-I-open* [?] *if $A = ic^*A$ and R-I-closed if $X - A$ is R-I-open.*

(2) *pre-I-open* [?] *if $A \subset ic^*A$ and pre-I-closed if $X - A$ is pre-I-open.*

It is clear that

regular open $\rightarrow$ R-I-open $\rightarrow$ open $\rightarrow$ pre-I-open $\rightarrow$ pre-open.

But the converse is not true, as shown by the following Example 2.3, Example 2.4 and Example 2.5.

Example 2.3. *Let $X = \{a, b, c, d\}$, $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}, X\}$ and $I = \{\emptyset, \{b\}\}$. Then*

$\tau(a) = \{\{a\}, \{a, b\}, \{a, b, c\}, X\}$, $\tau(b) = \{\{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}, X\}$, $\tau(c) = \{\{b, c\}, \{a, b, c\}, X\}$ and $\tau(d) = \{X\}$.

It is easy to prove that $c\{b\} = \{b, c, d\}$ and $\{b\}^* = \emptyset$. Thus $ic\{b\} = i\{b, c, d\} = \{b, c\} \neq \{b\}$ and $ic^*\{b\} = i\{b\} = \{b\}$. Hence $\{b\}$ is R-I-open in X but not regular open in X.

Example 2.4. *Let $X = \{a, b, c\}$, $\tau = \{\emptyset, \{b\}, \{b, c\}, X\}$ and $I = \{\emptyset, \{b\}\}$. Then $\tau(a) = \{X\}$, $\tau(b) = \{\{b\}, \{b, c\}, X\}$ and $\tau(c) = \{\{b, c\}, X\}$.*

(1) *Pick* $V = X \in \tau(a)$ *(resp.* $V = \{b\} \in \tau(b)$, $V = \{b, c\} \in \tau(c)$*), then* $\{b\} \cap V = \{b\} \in I$*, and so* $a \notin \{b\}^*$ *(resp.* $b \notin \{b\}^*$, $c \notin \{b\}^*$*). Thus* $\{b\}^* = \emptyset$.

For any $V \in \tau(a)$ *(resp.* $V \in \tau(c)$*),* $\{c\} \cap V = \{c\} \notin I$*, then* $a \in \{c\}^*$ *(resp.* $c \in \{c\}^*$*). Pick* $V = \{b\} \in \tau(b)$*, then* $\{c\} \cap V = \emptyset \in I$*, and so* $b \notin \{c\}^*$*. Thus* $\{c\}^* = \{a, c\}$*. This implies that* $c^*\{b, c\} = c^*\{b\} \cup c^*\{c\} = \{b\} \cup \{a, c\} = X$*. Hence* $ic^*\{b, c\} = X$.

Put $A = \{b, c\}$*, then* A *is open in* X*. But* $A \neq ic^*A$ *and so* A *is not R-I-open in* X.

(2) *For any* $V \in \tau(a)$ *(resp.* $V \in \tau(b)$, $V \in \tau(c)$*),* $\{b\} \cap V \neq \emptyset$*, then* $a \in \{b\}$ *(resp.* $b \in \{b\}$, $c \in \{b\}$*). Thus* $ic\{b\} = iX = X$ *and so* $ic\{a, b\} = X$.

For any $V \in \tau(a)$, $\{a\} \cap V = \{a\} \notin I$*, then* $a \in \{a\}^*$*. Pick* $V = \{b\} \in \tau(b)$*, then* $\{a\} \cap V = \emptyset \in I$*, and so* $b \notin \{a\}^*$*. Pick* $V = \{b, c\} \in \tau(c)$*, then* $\{a\} \cap V = \emptyset \in I$*, and so* $c \notin \{a\}^*$*. Thus* $\{a\}^* = \{a\}$*. This implies that* $c^*\{a, b\} = c^*\{a\} \cup c^*\{b\} = \{a\} \cup \{b\} = \{a, b\}$*. Hence* $ic^*\{a, b\} = i\{a, b\} = \{b\}$.

Put $B = \{a, b\}$*, then* $B \subset icB$*, and so* B *is pre-open in* X*. But* $B \not\subset ic^*B$ *and so* B *is not pre-I-open in* X.

Example 2.5. *Let* $X = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, \{a, c\}, X\}$ *and* $I = \{\emptyset, \{b\}\}$*. Then* $\tau(a) = \{\{a\}, \{a, c\}, X\}$, $\tau(b) = \{X\}$ *and* $\tau(c) = \{\{a, c\}, X\}$.

For any $V \in \tau(a)$ *(resp.* $V \in \tau(b)$, $V \in \tau(c)$*),* $\{a\} \cap V = \{a\} \notin I$*, then* $a \in \{a\}^*$ *(resp.* $b \in \{a\}^*$, $c \in \{a\}^*$*). Thus* $\{a\}^* = X$*, and so* $ic^*\{a\} = iX = X$*. Hence* $ic^*\{a, b\} = X$.

Put $A = \{a, b\}$*, then* $A \subset ic^*A$*, and so* A *is pre-I-open in* X*. But* A *is not open in* X.

Lemma 2.6. *Let* (X, τ, I) *be an ideal space. Then* ic^*A *is R-I-open in* X *for any* $A \subset X$.

Proof. Let $G = ic^*A$. $c^*G \supset G$ implies that $ic^*G \supset iG = G$. Now $ic^*A \subset c^*A$. Thus $ic^*G = ic^*ic^*A \subset ic^*c^*A = ic^*A = G$. Hence ic^*A is R-I-open in X. □

Definition 2.7 ([?])**.** *An ideal space* (X, τ, I) *is called* $*$*-extremally disconnected if the* $*$*-closure of each open subset of* X *is open in* X.

Definition 2.8. *A functions* $f : (X, \tau) \to (Y, \sigma)$ *is called almost continuous* [?]*(resp. weakly continuous* [?]*) if for each* $x \in X$ *and* $V \in \sigma(f(x))$*, there exists* $U \in \tau(x)$ *such that* $f(U) \subset icV$ *(resp.* $f(U) \subset cV$*).*

Definition 2.9 ([?])**.** *A functions* $f : (X, \tau) \to (Y, \sigma)$ *is called pre-continuous if* $f^{-1}(V)$ *is pre-open in* X *for each* $V \in \sigma$.

Definition 2.10 ([?])**.** *A functions* $f : (X, \tau, I) \to (Y, \sigma)$ *is called pre-I-continuous if* $f^{-1}(V)$ *is pre-I-open in* X *for each* $V \in \sigma$.

3. *I*-almost continuity

Definition 3.1. *A function* $f : (X, \tau) \to (Y, \sigma, I)$ *is called I-almost continuous if for each* $x \in X$ *and* $V \in \sigma(f(x))$*, there exists* $U \in \tau(x)$ *such that* $f(U) \subset ic^*V$.

Remark 3.2. (1) *Every I-almost continuous function is almost continuous.*

(2) *If $f:(X,\tau)\to(Y,\sigma)$ is continuous, then for every ideal I on Y, $f:(X,\tau)\to(Y,\sigma,I)$ is I-almost continuous.*

(3) *$f:(X,\tau)\to(Y,\sigma,I)$ is I-almost continuous if and only if $f:(X,\tau)\to(Y,\sigma^*)$ is almost continuous.*

(4) *If $I=\{\emptyset\}$, then $f:(X,\tau)\to(Y,\sigma,I)$ is I-almost continuous if and only if $f:(X,\tau)\to(Y,\sigma)$ is almost continuous.*

Remark 3.3. (1) *A subset A of a space (X,τ,I) is called almost-I-open* [**?**] *if $A\subset ciA^*$.*

(2) *Abd El-Monsef et al.* [**?**] *introduced the concept of almost I-continuity, where a function $f:(X,\tau,I)\to(Y,\sigma)$ is called almost-I-continuous if $f^{-1}(V)$ is almost-I-open in X for any $V\in\sigma$.*

Example 3.4. *Let $X=\{a,b,c\}$, $\tau=\{\emptyset,\{a\},\{c\},\{a,c\},X\}$, $\sigma=\{\emptyset,\{b\},\{b,c\},X\}$ and $I=\{\emptyset,\{b\}\}$. Then, the identity function $f:(X,\tau)\to(X,\sigma,I)$ is almost continuous but not I-almost continuous.*

(1) *Since $\sigma(a)=\{X\}$ (resp. $\sigma(b)=\{\{b\},\{b,c\},X\}$, $\sigma(c)=\{\{b,c\},X\}$), then for any $V\in\sigma(a)$ (resp. for any $V\in\sigma(b)$, for any $V\in\sigma(c)$), $V\cap\{b\}\neq\emptyset$. Thus $c\{b\}=X$. So $ic\{b\}=X$. For any $x\in X$ and $V\in\sigma(f(x))=\sigma(x)$, we have $\{b\}\subset V$. This implies that $icV=X$. Hence there exists $U\in\tau(x)$ such that $f(U)=U\subset icV$. Therefore f is almost continuous.*

(2) *Pick $V=X\in\sigma(a)$ (resp. $V=\{b\}\in\sigma(b)$, $V=\{b,c\}\in\sigma(c)$), then $\{b\}\cap V=\{b\}\in I$, and so $a\notin\{b\}^*$ (resp. $b\notin\{b\}^*$, $c\notin\{b\}^*$). This implies that $\{b\}^*=\emptyset$.*

Let $V=\{b\}\in\sigma(f(b))=\sigma(b)$. For any $U\in\tau(b)$, since $\tau(b)=\{X\}$, then $U=X$. Thus $f(U)=f(X)=X\not\subset ic^*\{b\}=i\{b\}=\{b\}$. Therefore f is not I-almost continuous.

Example 3.5. *Let $X=\{a,b,c\}$, $\tau=\{\emptyset,\{a\},\{b\},\{a,b\},X\}$, $\sigma=\{\emptyset,\{b\},\{b,c\},X\}$ and $I=\{\emptyset,\{b\}\}$. Then, the identity function $f:(X,\tau)\to(X,\sigma,I)$ is I-almost continuous but not continuous.*

(1) *Now $\tau(c)=\{X\}$ and $\sigma(c)=\{\{b,c\},X\}$). Pick $V=\{b,c\}\in\sigma(f(c))=\sigma(c)$, then for any $U\in\tau(c)$, $f(U)=f(X)=X\not\subset V$. Hence f is not continuous.*

(2) *By Example 3.4, $\{b\}^*=\emptyset$. Now $\sigma(a)=\{X\}$, $\sigma(b)=\{\{b\},\{b,c\},X\}$, $\sigma(c)=\{\{b,c\},X\}$). For any $V\in\sigma(a)$ (resp. $V\in\sigma(c)$), $\{c\}\cap V=\{c\}\notin I$, then $a\in\{c\}^*$ (resp. $c\in\{c\}^*$). Pick $V=\{b\}\in\sigma(b)$, then $\{c\}\cap V=\emptyset\in I$, and so $b\notin\{c\}^*$. Thus $\{c\}^*=\{a,c\}$. This implies that $c^*\{b,c\}=c^*\{b\}\cup c^*\{c\}=\{b\}\cup\{a,c\}=X$.*

(*a*) *For any $V\in\sigma(f(a))=\sigma(a)$ (resp. $V\in\sigma(f(c))=\sigma(c)$), $\{b,c\}\subset V$ implies $ic^*V=iX=X$, then there exists $U\in\tau(a)$ (resp. $U\in\tau(c)$)such that $f(U)=U\subset ic^*V$.*

(*b*) *For any $V\in\sigma(f(b))=\sigma(b)$, if $V=\{b\}$, then there exists $U=\{b\}\in\tau(b)$ such that $f(U)=U\subset ic^*V=\{b\}$; if $V=\{b,c\}$ or if $V=X$, then there exists $U\in\tau(b)$ such that $f(U)=U\subset ic^*V=X$.*

By (a) and (b), f is I-almost continuous.

The following Theorem 3.6 and Theorem 3.8 give some characterizations of I-almost continuity.

Theorem 3.6. *Let $f : (X, \tau) \to (Y, \sigma, I)$ be a function. Then the following are equivalent.*

(1) *f is I-almost continuous.*

(2) *$f^{-1}(V) \subset if^{-1}(ic^*V)$ for any $V \in \sigma$.*

(3) *$f^{-1}(F) \supset cf^{-1}(ci^*F)$ for any closed subset F of Y.*

(4) *$f^{-1}(cB) \supset cf^{-1}(ci^*cB)$ for any $B \subset Y$.*

(5) *$f^{-1}(iB) \subset if^{-1}(ic^*iB)$ for any $B \subset Y$.*

(6) *For any R-I-open subset G of Y, $f^{-1}(G) \in \tau$.*

(7) *For any R-I-closed subset F of Y, $f^{-1}(F)$ is closed in X.*

(8) *For any $V \in \sigma$, $f^{-1}(ic^*V) \in \tau$.*

(9) *For any $x \in X$ and R-I-open subset V of Y with $f(x) \in V$, there exists $U \in \tau(x)$ such that $f(U) \subset V$.*

Proof. (1) $\Rightarrow$ (2). Suppose $f^{-1}(V) - if^{-1}(ic^*V) \neq \emptyset$ for some $V \in \sigma$. Pick $x \in f^{-1}(V) - if^{-1}(ic^*V)$. $x \in f^{-1}(V)$ implies that $V \in \sigma(f(x))$. Since f is I-almost continuous, then there exists $U \in \tau(x)$ such that $f(U) \subset ic^*V$. Thus $U \subset f^{-1}(ic^*V)$. So $x \in if^{-1}(ic^*V)$, a contradiction.

(2) $\Rightarrow$ (3). For any closed subset F of Y, $Y - F \in \sigma$. By (2), $f^{-1}(Y - F) \subset if^{-1}(ic^*(Y - F))$. Now, $f^{-1}(Y - F) = X - f^{-1}(F)$

and

$$\begin{aligned} &if^{-1}(ic^*(Y - F)) = if^{-1}(i(Y - i^*F)) \\ &= if^{-1}(Y - ci^*F)) = i(X - f^{-1}(ci^*F) \\ &= X - cf^{-1}(ci^*F). \end{aligned}$$

Hence $f^{-1}(F) \supset cf^{-1}(ci^*F)$.

(3) $\Rightarrow$ (4) is obvious.

(4) $\Rightarrow$ (5). For any $B \subset Y$, by (4), $f^{-1}(c(Y - B)) \supset cf^{-1}(ci^*c(Y - B))$. Now $f^{-1}(c(Y - B)) = f^{-1}(Y - iB) = X - f^{-1}(iB)$

and

$$\begin{aligned} &cf^{-1}(ci^*c(Y - B)) = cf^{-1}(ci^*(Y - iB)) \\ &= cf^{-1}(c(Y - c^*iB)) = cf^{-1}(Y - ic^*iB) \\ &= c(X - f^{-1}(ic^*iB)) = X - if^{-1}(ic^*iB). \end{aligned}$$

Hence $f^{-1}(iB) \subset if^{-1}(ic^*iB)$.

(5) $\Rightarrow$ (6). For any R-I-open subset G of Y, by (5), $f^{-1}(iG) \subset if^{-1}(ic^*iG)$. Now $iG = G$ and $ic^*iG = ic^*G = G$.

Thus $f^{-1}(G) \subset if^{-1}(G)$. This implies that $f^{-1}(G) \in \tau$.

(6) $\Rightarrow$ (1). For each $x \in X$ and $V \in \sigma(f(x))$, let $G = ic^*V$, By Lemma 2.6, G is R-I-open subset G of Y. By (6), $f^{-1}(G) \in \tau$. Put $U = f^{-1}(G)$, then $U \in \tau(x)$ such that $f(U) \subset ic^*V$. Hence f is I-almost continuous.

(6) $\Leftrightarrow$ (7) is clear.

(6) $\Rightarrow$ (8) holds by Lemma 2.6.

(8) $\Rightarrow$ (2) is obvious.

(6) $\Rightarrow$ (9). For any $x \in X$ and R-I-open subset V of Y with $V \in \sigma(f(x))$, by (6), $f^{-1}(V) \in \tau$. Put $U = f^{-1}(V)$, then $U \in \tau(x)$ such that $f(U) \subset V$.

(9) $\Rightarrow$ (6). Let G be R-I-open in Y. For any $x \in f^{-1}(G)$. $f(x) \in G$, by (6), there exists $U \in \tau(x)$ such that $f(U) \subset G$. This implies that $U \subset f^{-1}(G)$. Thus $f^{-1}(G) \in \tau$. □

If $I = \{\emptyset\}$ in Theorem 3.6, then we have the following Corollary 3.7.

Corollary 3.7. *Let $f : (X, \tau) \to (Y, \sigma)$ be a function. Then the following are equivalent.*

(1) *f is almost continuous.*
(2) *$f^{-1}(V) \subset if^{-1}(icV)$ for any $V \in \sigma$.*
(3) *$f^{-1}(icV) \in \tau$ for any $V \in \sigma$.*
(4) *$f^{-1}(cB) \supset cf^{-1}(cicB)$ for any $B \subset Y$.*
(5) *$f^{-1}(iB) \subset if^{-1}(iciB)$ for any $B \subset Y$.*
(6) *For any regular-open subset G of Y, $f^{-1}(G) \in \tau$.*

Theorem 3.8. *A function $f : (X, \tau) \to (Y, \sigma, I)$ is I-almost continuous if and only if for any pre-I-open subset G of Y, $f^{-1}(G) \subset if^{-1}(ic^*G)$.*

Proof. Necessity. Let G be any pre-I-open subset of Y, then $G \subset ic^*G$, and so $f^{-1}(G) \subset f^{-1}(ic^*G)$. By Lemma 2.6 , ic^*G is R-I-open in Y. Since f is I-almost continuous, then $f^{-1}(G) \in \tau$ by Theorem 3.6. Thus

$$f^{-1}(G) \subset f^{-1}(ic^*G) = if^{-1}(ic^*G).$$

Sufficiency. Let G be any R-I-open subset of Y, then G is pre-I-open in Y. By hypothesis,

$$f^{-1}(G) \subset if^{-1}(ic^*G) = if^{-1}(G).$$

Thus $f^{-1}(G) \in \tau$. By Theorem 3.6, f is I-almost continuous. □

4. *I*-almost continuity and other forms of continuity

Definition 4.1 ([?]). *A function $f : (X, \tau) \to (Y, \sigma, I)$ is called weakly I-continuous, if for each $x \in X$ and $V \in \sigma(f(x))$, there exists $U \in \tau(x)$ such that $f(U) \subset c^*V$.*

Remark 4.2. *Every I-almost continuous function is weakly I-continuous and every weakly I-continuous function is weakly continuous.*

Example 4.3. *Let $X = \{a, b, c, d\}$, $\tau = \{\emptyset, \{a\}, \{a, c\}, \{a, b, d\}, X\}$,*
$\sigma = \{\emptyset, \{c\}, \{b, d\}, \{b, c, d\}, X\}$ and $I = \{\emptyset, \{b\}\}$.

Then, the identity function $f : (X, \tau) \to (X, \sigma, I)$ is weakly I-continuous but not I-almost continuous.

(1) *Since $\sigma(a) = \{X\}$, $\sigma(b) = \{\{b, d\}, \{b, c, d\}, X\}$, $\sigma(c) = \{\{c\}, \{b, c, d\}, X\}$ and $\sigma(d) = \{\{b, d\}, \{b, c, d\}, X\}$), then $\{a\}^* = \{a\}$, $\{b\}^* = \emptyset$, $\{c\}^* = \{a, c\}$ and $\{d\}^* = \{a, b, d\}$. Thus*

$c^*\{c\} = \{a, c\}$, $c^*\{b, d\} = \{a, b, d\}$ and $c^*\{b, c, d\} = X$.

(a) *For any $V \in \sigma(f(a)) = \sigma(a)$, $V = X$, it is obvious that $f(U) = U \subset c^*V$ for some $U \in \tau(a)$.*

(b) *For any $V \in \sigma(f(b)) = \sigma(b)$, if $V = \{b, d\}$, pick $U = \{a, b, d\} \in \tau(b)$, then $f(U) = U \subset c^*V$; if $V = \{b, c, d\}$ or $V = X$, it is obvious that $f(U) = U \subset c^*V = X$ for some $U \in \tau(b)$.*

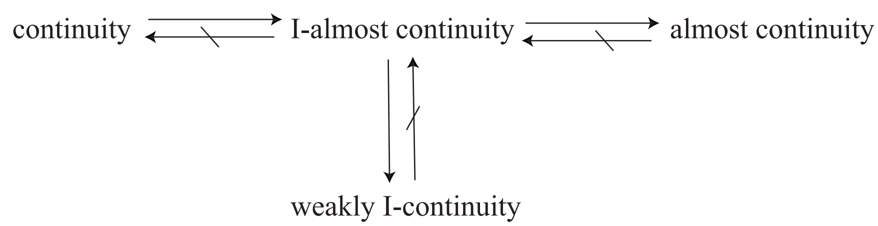
(c) For any $V \in \sigma(f(c)) = \sigma(c)$, if $V = \{c\}$, pick $U = \{a, c\} \in \tau(c)$, then $f(U) = U \subset c^*V$; if $V = \{b, c, d\}$ or $V = X$, it is obvious that $f(U) = U \subset c^*V$ for some $U \in \tau(c)$.

(d) For any $V \in \sigma(f(d)) = \sigma(d)$, if $V = \{b, d\}$, pick $U = \{a, b, d\} \in \tau(d)$, then $f(U) = U \subset c^*V$; if $V = \{b, c, d\}$ or $V = X$, it is obvious that $f(U) = U \subset c^*V$ for some $U \in \tau(d)$.

Hence f is weakly I-continuous.

(2) Pick $V = \{b, d\} \in \sigma(f(b)) = \sigma(b)$, then $ic^*V = i\{a, b, d\} = \{b, d\}$. For any $U \in \tau(b)$, $U = \{a, b, d\}$ or $U = X$, then $f(U) = U \not\subset ic^*V$. Hence f is not I-almost continuous.

From Example 3.4, Example 3.5 and Example 4.3, we have the following relationships:



The following Theorem 4.4 improve Theorem 2.2 in [**?**].

Theorem 4.4. *Let $f : (X, \tau) \to (Y, \sigma, I)$ be a function. Then the following are equivalent.*

(1) f is weakly I-continuous.
(2) $f^{-1}(V) \subset if^{-1}(c^*V)$ for any $V \in \sigma$.
(3) $f^{-1}(F) \supset cf^{-1}(i^*F)$ for any closed subset F of Y.
(4) $f^{-1}(cB) \supset cf^{-1}(i^*cB)$ for any $B \subset Y$.
(5) $f^{-1}(iB) \subset if^{-1}(c^*iB)$ for any $B \subset Y$.

Proof. (1) $\Rightarrow$ (2). Suppose $f^{-1}(V) - if^{-1}(c^*V) \neq \emptyset$ for some $V \in \sigma$. Pick $x \in f^{-1}(V) - if^{-1}(c^*V)$. $x \in f^{-1}(V)$ implies that $V \in \sigma(f(x))$. Since f is weakly I-continuous, then there exists $U \in \tau(x)$ such that $f(U) \subset c^*V$. Thus $U \subset f^{-1}(c^*V)$. So $x \in if^{-1}(c^*V)$, a contradiction.

(2) $\Rightarrow$ (3). For any closed subset F of Y, $Y - F \in \sigma$. By hypothesis, $f^{-1}(Y - F) \subset if^{-1}(c^*(Y - F))$. Now, $f^{-1}(Y - F) = X - f^{-1}(F)$

and

$$if^{-1}(c^*(Y - F)) = X - cf^{-1}(i^*F).$$

Hence $f^{-1}(F) \supset cf^{-1}(i^*F)$.

(3) $\Rightarrow$ (4) is obvious.

(4) $\Rightarrow$ (5). For any $B \subset Y$, by hypothesis, $f^{-1}(c(Y - B)) \supset cf^{-1}(i^*c(Y - B))$. Now $f^{-1}(c(Y - B)) = f^{-1}(Y - iB) = X - f^{-1}(iB)$

and

$$cf^{-1}(i^*c(Y - B)) = X - if^{-1}(c^*iB).$$

Hence $f^{-1}(iB) \subset if^{-1}(c^*iB)$.

(5) $\Rightarrow$ (2) is obvious.

(2) $\Rightarrow$ (1). For any $x \in X$ and $V \in \sigma(f(x))$, since $f(x) \in V$, then $x \in f^{-1}(V)$. By hypothesis, $f^{-1}(V) \subset if^{-1}(c^*V)$. Thus $x \in if^{-1}(c^*V)$. This implies that $U \subset f^{-1}(c^*V)$ for some $U \in \tau(x)$. So $f(U) \subset c^*V$. Hence f is weakly I-continuous. □

If $I = \{\emptyset\}$ in Theorem 4.4, then we have the following Corollary 4.5.

Corollary 4.5. *Let $f : (X, \tau) \to (Y, \sigma)$ be a function. Then the following are equivalent.*

(1) *f is weakly continuous.*
(2) *$f^{-1}(V) \subset if^{-1}(cV)$ for any $V \in \sigma$.*
(3) *$f^{-1}(F) \supset cf^{-1}(i^*F)$ for any closed subset F of Y.*
(4) *$f^{-1}(cB) \supset cf^{-1}(icB)$ for any $B \subset Y$.*
(5) *$f^{-1}(iB) \subset if^{-1}(ciB)$ for any $B \subset Y$.*

Theorem 4.6. *Let $f : (X, \tau) \to (Y, \sigma, I)$ be a function. If f is a weakly I-continuous and open function, then f is I-almost continuous.*

Proof. For any $V \in \sigma$, since f is weakly I-continuous, then $f^{-1}(V) \subset if^{-1}(c^*V)$ by Theorem 4.4. Now f is open, by Theorem 1.5.2 in [**?**], $f^{-1}(cB) \subset cf^{-1}(B)$ for any $B \subset Y$. Thus, $X - f^{-1}(ic^*V) = f^{-1}(Y - ic^*V) = f^{-1}(c(Y - c^*V) \subset cf^{-1}(Y - c^*V) = c(X - f^{-1}(c^*V)) = X - if^{-1}(c^*V)$. This implies that $if^{-1}(c^*V) \subset f^{-1}(ic^*V)$. So $if^{-1}(c^*V) = iif^{-1}(c^*V) \subset if^{-1}(ic^*V)$. Hence $f^{-1}(V) \subset if^{-1}(ic^*V)$. By Theorem 3.6, f is I-almost continuous. □

Theorem 4.7. *Let $f : (X, \tau) \to (Y, \sigma, I)$ be a function. If f is pre-continuous such that $cf^{-1}(V) \subset f^{-1}(ic^*V)$ for any $V \in \sigma$, then f is I-almost continuous.*

Proof. For any $V \in \sigma$, by hypothesis, $cf^{-1}(V) \subset f^{-1}(ic^*V)$. Since f is pre-continuous, then $f^{-1}(V)$ is pre-open in X, so $f^{-1}(V) \subset icf^{-1}(V)$. Thus $f^{-1}(V) \subset if^{-1}(ic^*V)$. By Theorem 3.6, f is I-almost continuous. □

Lemma 4.8 ([**?**]). *If (X, τ, I) is an ideal space, (Y, σ) is a space and $f : (X, \tau) \to (Y, \sigma)$ is a map, then $f(I) = \{f(A) : A \in I\}$ is an ideal on Y.*

Theorem 4.9. *Let $f : (X, \tau, I) \to (Y, \sigma, J)$ be a function where $J = f(I)$. If f is pre-I-continuous such that $c^*f^{-1}(V) \subset f^{-1}(ic^*V)$ for any $V \in \sigma$, then f is J-almost continuous.*

Proof. For any $V \in \sigma$, by hypothesis, $c^*f^{-1}(V) \subset f^{-1}(ic^*V)$. Since f is pre-I-continuous, then $f^{-1}(V)$ is pre-I-open in X, so $f^{-1}(V) \subset ic^*f^{-1}(V)$. Thus $f^{-1}(V) \subset if^{-1}(ic^*V)$. By Theorem 3.6, f is J-almost continuous. □

We can easily prove the following Theorem 4.10.

Theorem 4.10. *Let $f : (X, \tau) \to (Y, \sigma, I)$ be a function such that Y is $*$-extremally disconnected. Then f is I-almost continuous if and only if f is weakly I-continuous.*

Theorem 4.11. *If Y is regular and $f : (X, \tau) \to (Y, \sigma, I)$ is I-almost continuous, then f is continuous.*

Proof. Let $V \in \sigma$. For each $y \in V$, since Y is regular, then $y \in W_y \subset cW_y \subset V$ for some $W_y \in \sigma$, and so $y \in W_y \subset ic^*W_y \subset cW_y \subset U$. Thus,

$$V = \bigcup\{W_y : y \in V\} = \bigcup\{ic^*W_y : y \in V\},$$

and

$$f^{-1}(V) = \bigcup\{f^{-1}(W_y) : y \in V\} = \bigcup\{f^{-1}(ic^*W_y) : y \in V\}.$$

Now f is I-almost continuous, by Theorem 3.6, $f^{-1}(W_y) \subset if^{-1}(ic^*W_y)$ for any $y \in V$. Thus

$$\begin{aligned} f^{-1}(V) &= \bigcup\{f^{-1}(W_y) : y \in V\} \subset \bigcup if^{-1}(ic^*W_y) : y \in V\} \\ &\subset i(\bigcup\{f^{-1}(ic^*W_y) : y \in V\}) = if^{-1}(V). \end{aligned}$$

This implies that $f^{-1}(V)$ is open in X. Hence f is continuous. □

Let (X, τ) be a space and $A \subset X$. The semi-closure [**?**] of A, denoted by c_sA, is defined as the intersection of all semi-closed subsets containing A. A is semi-closed if and only if $c_sA = A$ [**?**]. It was proved that $c_sA = A \cup icA$ for $A \in \tau$ in [**?**]. The semi-boundary of A, denoted by ∂_sA, is defined as $c_sA \cap c_s(X - A)$.

Definition 4.12. *Let (X, τ, I) be an ideal space and $A \subset X$. Then*
(1) *$A \cup ic^*A$ is called the semi-$*$-closure of A, which is denoted by c^*_sA.*
(2) *$c^*_sA \cap c^*_s(X - A)$ is called the semi-$*$-boundary of A, which is denoted by ∂^*_sA.*
(3) *$\mathcal{F}^*_sA = X - \partial^*_sA$.*

Definition 4.13. *We call that a function $f : (X, \tau) \to (Y, \sigma, I)$ satisfies the semi-$*$-boundary condition, and denote it by ∂^*_s-condition, if $f^{-1}(\partial^*_sV)$ is closed in X for any $V \in \sigma$.*

Lemma 4.14. *Let (X, τ, I) be an ideal space. If $U \in \tau$, then*
(1) *$\partial^*_sU = ic^*U - U$.*
(2) *$\mathcal{F}^*_sU = U \cup (X - ic^*U)$.*

Proof. (1) Since $U \subset ic^*U$, then $c^*_sU = U \cup ic^*U = ic^*U$. Now $i^*U \supset iU = U$ implies that $ci^*U \supset cU \supset U$. Thus $c^*_s(X-U) = (X-U) \cup ic^*(X-U) = (X-U) \cup (X-ci^*U) = X - U \cap ci^*U = X - U$.
Hence $\partial^*_sU = c^*_sU \cap c^*_s(X - U) = ic^*U \cap (X - U) = ic^*U - U$.
(2) By (1), $\partial^*_sU = ic^*U - U$. Hence $\mathcal{F}^*_sU = X - (ic^*U - U) = U \cup (X - ic^*U)$. □

Theorem 4.15. *If $f : (X, \tau) \to (Y, \sigma, I)$ is I-almost continuous and satisfies the ∂^*_s-condition, then f is continuous.*

Proof. Let $V \in \sigma$. By hypothesis, $f^{-1}(\partial^*_sV)$ is closed in X. Since f is I-almost continuous and ic^*V is R-I-open in Y, then $f^{-1}(ic^*V) \in \tau$ by Theorem 3.6. By Lemma 4.14,

$$\begin{aligned} f^{-1}(V) &= f^{-1}(ic^*V \cap (V \cup (Y - ic^*V))) \\ &= f^{-1}(ic^*V \cap (Y - \partial^*_sV)) \\ &= f^{-1}(ic^*V) \cap (X - f^{-1}(\partial^*_sV)). \end{aligned}$$

Thus $f^{-1}(V) \in \tau$. Hence f is continuous. □

5. Properties of I-almost continuity

We can easily prove the following Theorem 5.1.

Theorem 5.1. *Let $f:(X,\tau)\to(Y,\sigma,I)$ and $g:(X,\tau)\to(Y,\sigma,J)$ be two functions where $I\subset J$. If g is J-almost continuous, then f is I-almost continuous.*

Theorem 5.2. *If $f:(X,\tau)\to(Y,\sigma)$ is continuous and $g:(Y,\sigma)\to(Z,\varphi,I)$ is I-almost continuous, then $g\circ f:(X,\tau)\to(Z,\varphi,I)$ is I-almost continuous.*

Proof. Let G be any R-I-open subset of Z. Since g is I-almost continuous, then $f^{-1}(G)\in\sigma$ by Theorem 3.6. Because f is continuous, thus $(g\circ f)^{-1}(G)=g^{-1}(f^{-1}(G))\in\tau$. By Theorem 3.6, $g\circ f$ is I-almost continuous. □

Theorem 5.3. *Let $f:(X,\tau)\to(Y,\sigma,I)$ be a function. Then f is I-almost continuous if and only if the restriction $f\mid_Z:(Z,\tau_Z)\to(Y,\sigma,I)$ is I-almost continuous for any $Z\subset X$.*

Proof. "Sufficiency" is obvious, we will prove "Necessity". Denote $g=f\mid_Z$. For any $x\in Z$ and $V\in\sigma(g(x))$, $x\in Z$ implies that $g(x)=f(x)$, then $V\in\sigma(f(x))$. Since f is I-almost continuous, then there exists $U\in\tau(x)$ such that $f(U)\subset ic^*V$. Now $U\cap Z\in\tau_Z(x)$, $g(U\cap Z)=f(U\cap Z)\subset ic^*V$. Thus g is I-almost continuous. □

If I is an ideal of (X,τ) and $Y\subset X$, then I_Y is an ideal of (Y,τ_Y) [?]. So (Y,τ_Y,I_Y) is also an ideal space. Thus, (Y,τ_Y,I_Y) is called subspace of (X,τ,I). If $A\subset Y$, then the closure of A in (Y,τ_Y), the interior of A in (Y,τ_Y), the closure of A in (Y,τ_Y,I_Y) and the interior of A in (Y,τ_Y,I_Y) denote by c_YA, i_YA, c^*_YA and i^*_YA, respectively.

Lemma 5.4. *Let (X,τ,I) be an ideal space. Then*
(1) *If $A\subset Y\subset X$, then $A^*(I_Y,\tau_Y)=A^*(I,\tau)\bigcap Y$.*
(2) *If $A\subset Y\subset X$ and Y is open in X, then $i_Yc^*_YA=(ic^*A)\cap Y$.*

Proof. (1) Since $A^*(I_Y,\tau_Y)=\{y\in Y: V\cap A\notin I_Y \text{ for every } V\in\tau_Y(y)\}$, then $A^*(I_Y,\tau_Y)\subset Y$. If $y\in A^*(I_Y,\tau_Y)$, then for every $U\in\tau(y)$, $U\cap Y\in\tau_Y(y)$, so $(U\cap Y)\cap A\notin I_Y$. Obviously, $(U\cap Y)\cap A=U\cap A$. Thus, $U\cap A\notin I$. This implies $y\in A^*(I,\tau)$. Hence $A^*(I_Y,\tau_Y)\subset A^*(I,\tau)\bigcap Y$.

On the other hands. If $y\in A^*(I,\tau)\cap Y$, for every $V\in\tau_Y(y)$, then there exists $U\in\tau$ such that $V=U\cap Y$, so $U\in\tau(y)$. This implies $U\cap A\notin I$. Obviously, $I_Y\subset I$, $U\cap Y\cap A=U\cap A$. Thus, $V\cap A=U\cap Y\cap A\notin I_Y$. So $y\in A^*(I_Y,\tau_Y)$. Hence $A^*(I,\tau)\bigcap Y\subset A^*(I_Y,\tau_Y)$.

(2) By (1),
$$\begin{aligned}c^*_YA&=A^*(I_Y,\tau_Y)\cup A=(A^*(I,\tau)\cap Y)\cup A\\&=(A^*(I,\tau)\cup A)\cap(Y\cup A)=c^*A\cap Y.\end{aligned}$$
Thus,
$$\begin{aligned}i_Yc^*_YA&=ic^*_YA\cap Y=i(c^*A\cap Y)\cap Y\\&=(ic^*A\cap iY)\cap Y=(ic^*A)\cap Y.\end{aligned}$$
□

Lemma 5.5 ([?]). *Let $\{(X_\alpha,\tau_\alpha,I_\alpha):\alpha\in\Lambda\}$ be a family of pairwise disjoint ideal spaces. Then $\{\bigcup_{\alpha\in\bigwedge} I_\alpha : I_\alpha\in I_\alpha\}$ is a ideal on $\bigcup_{\alpha\in\bigwedge} X_\alpha$.*

Let $\{(X_\alpha, \tau_\alpha) : \alpha \in \Lambda\}$ and $\{(Y_\alpha, \sigma_\alpha, I_\alpha) : \alpha \in \Lambda\}$ be two families of pairwise disjoint spaces, i.e., $X_\alpha \cap X'_\alpha = Y_\alpha \cap Y'_\alpha = \emptyset$ if $\alpha \neq \alpha'$, and let $f_\alpha : (X_\alpha, \tau_\alpha) \to (Y_\alpha, \sigma_\alpha, I_\alpha)$ be a function for each $\alpha \in \Lambda$. We denote the topological sum $(\bigcup_{\alpha\in\bigwedge} X_\alpha, \tau)$ of $\{(X_\alpha, \tau_\alpha) : \alpha \in \Lambda\}$ by $\bigoplus_{\alpha\in\bigwedge} X_\alpha$ and the topological sum $(\bigcup_{\alpha\in\bigwedge} Y_\alpha, \sigma, I)$ of $\{(Y_\alpha, \sigma_\alpha, I_\alpha) : \alpha \in \Lambda\}$ by $\bigoplus_{\alpha\in\bigwedge} Y_\alpha$, where

$$\tau = \{A \subset X : A \cap X_\alpha \in \tau_\alpha \ for\ every\ \alpha \in \Lambda\},$$
$$\sigma = \{B \subset Y : B \cap Y_\alpha \in \sigma_\alpha \ for\ every\ \alpha \in \Lambda\},$$

and

$$I = \{\bigcup_{\alpha\in\bigwedge} A_\alpha : A_\alpha \in I_\alpha\}.$$

A function $\bigoplus_{\alpha\in\bigwedge} f_\alpha : \bigoplus_{\alpha\in\bigwedge} X_\alpha \to \bigoplus_{\alpha\in\bigwedge} Y_\alpha$, called a sum function of $\{f_\alpha : \alpha \in \Lambda\}$, is defined as follows: for every $x \in \bigcup_{\alpha\in\bigwedge} X_\alpha$,

$$(\bigoplus_{\alpha\in\bigwedge} f_\alpha)(x) = f_\beta(x) \text{ if there exists unique } \beta \in \bigwedge \text{ such that } x \in X_\beta.$$

Theorem 5.6. *Let* $\bigoplus_{\alpha\in\bigwedge} X_\alpha = (\bigcup_{\alpha\in\bigwedge} X_\alpha, \tau)$ *and* $\bigoplus_{\alpha\in\bigwedge} Y_\alpha = (\bigcup_{\alpha\in\bigwedge} Y_\alpha, \sigma, I)$*. Then* $\bigoplus_{\alpha\in\bigwedge} f_\alpha : \bigoplus_{\alpha\in\bigwedge} X_\alpha \to \bigoplus_{\alpha\in\bigwedge} Y_\alpha$ *is I-almost continuous if and only if* $f_\alpha : (X_\alpha, \tau_\alpha) \to (Y_\alpha, \sigma_\alpha, I_\alpha)$ *is I_α-almost continuous for every* $\alpha \in \bigwedge$.

Proof. Denote $f = \bigoplus_{\alpha\in\bigwedge} f_\alpha$, $X = \bigoplus_{\alpha\in\bigwedge} X_\alpha$, $Y = \bigoplus_{\alpha\in\bigwedge} Y_\alpha$.

Necessity. For every $\alpha \in \Lambda$, $I_{X_\alpha} = I_\alpha$ is obvious. We will prove that f_α is I_α-almost continuous. Let $x \in X_\alpha$ and $V \in \sigma_\alpha(f_\alpha(x))$. Since Y_α is open in Y and $f(x) = f_\alpha(x)$, then $V \in \sigma(f(x))$. By f is I-almost continuous, there exists $U \in \tau(x)$ such that $f(U) \subset ic^*V$. Now $U \cap X_\alpha \in \tau_\alpha(x)$, by Lemma 5.4,

$$f_\alpha(U \cap X_\alpha) = f(U \cap X_\alpha) \subset f(U) \cap Y_\alpha \subset ic^*V \cap Y_\alpha = i_{Y_\alpha} c^*_{Y_\alpha} V.$$

Hence f_α is I_α-almost continuous.

Sufficiency. Let $x \in X$ and $V \in \sigma(f(x))$. Then there exists unique $\beta \in \bigwedge$ such that $x \in X_\beta$. Now $f(x) = f_\beta(x)$ and $V \cap Y_\beta \in \sigma_\beta(f_\beta(x))$. Since f_β is I_β-almost continuous, then there exists $U \in \tau_\beta(x)$ such that $f_\beta(U) \subset i_{Y_\beta} c^*_{Y_\beta}(V \cap Y_\beta)$. Since X_β is open in X, then $U \in \tau(x)$. Now Y_β is open in Y, by Lemma 5.4,

$$f(U) = f_\beta(U) \subset ic^*(V \cap Y_\beta) \cap Y_\beta \subset ic^*V.$$

Hence f is I-almost continuous. □

6. New decomposition of continuity with I-almost continuity as a factor

Some weak forms of continuity such as almost continuity, I-almost continuity, weak continuity and weak I-continuity are given in terms of the operators of interior, closure, boundary, etc. In order to give a general approach to weak forms of continuity and a general setting for decomposition of continuity, Tong [**?**] introduced the concepts of expansion on open sets, mutually dual expansion and expansion-continuity. We obtain new decomposition of continuity with I-almost continuity as a factor by means of these concepts.

Definition 6.1 ([**?**])**.** *Let (X,τ) be a space. A function $\mathcal{A}:(X,\tau)\to 2^X$ is called an expansion on (X,τ) if $U\subset \mathcal{A}U$ for each $U\in\tau$.*

Expansions are easily found. For instance $\mathcal{A}=i,c,ic,ci,ic^*,ci^*,cic$.

Definition 6.2 ([**?**])**.** *Let (X,τ) be a space. A pair of expansions $\mathcal{A}$, $\mathcal{B}$ on (X,τ) is called mutually dual if $\mathcal{A}U\cap\mathcal{B}U=U$ for each $U\in\tau$.*

Remark 6.3. *i-expansion is mutually dual to any expansion $\mathcal{A}$.*

Definition 6.4 ([**?**])**.** *Let (X,τ) and (Y,σ) be two spaces, $\mathcal{A}$ an expansion on (Y,σ). A function $f:(X,\tau)\to(Y,\sigma)$ is called $\mathcal{A}$-expansion continuous if $f^{-1}(V)\subset if^{-1}(\mathcal{A}V)$ for each $V\in\sigma$.*

Obviously, continuity is equivalent to i-expansion continuity.

Theorem 6.5. *Let I be a ideal on Y. Then a function $f:(X,\tau)\to(Y,\sigma)$ is continuous if and only if $f:(X,\tau)\to(Y,\sigma,I)$ is I-almost continuous and $\mathcal{F}_s^*$-expansion continuous.*

Proof. "Necessity" is obvious, we will prove "Sufficiency". Let $V\in\sigma$. Since f is I-almost continuous, then $f^{-1}(V)\subset if^{-1}(ic^*V)$ by Theorem 3.6. Now f is $\mathcal{F}_s^*$-expansion continuous. Thus $f^{-1}(V)\subset if^{-1}(\mathcal{F}_s^*V)$. By Lemma 4.14, $\mathcal{F}_s^*V=V\cup(X-ic^*V)$. So ic^*-expansion continuity and $\mathcal{F}_s^*$-expansion continuity are mutually dual. This implies that $f^{-1}(V)\subset if^{-1}(ic^*V)\cap if^{-1}(\mathcal{F}_s^*V)=if^{-1}(ic^*V\cap\mathcal{F}_s^*V)=if^{-1}(V)$. Thus $f^{-1}(V)\in\tau$. Hence f is continuous. □

References

1. M.E.Abd El-Monsef, R.A.Mahmoud, A.A.Nasef, Almost I-openness and almost-I-continuity, J. Egyptian Math. Soc., 7(1999), 191-200.
2. A.Acikgoz, S.Yuksel, Decompositions of new classes of functions, Chaos, Solitons and Fractals, 40(2009), 408-413.
3. S.G.Crossle, S.K.Hildebrand, Semi-closure, Texas J. Sci., 22(1971), 99-112.
4. S.G.Crossle, S.K.Hildebrand, Semi-topological properties, Fund. Math., 74(1972), 233-254.
5. J.Dontchev, On Hausdorff spaces via topological ideals and I-irresolute functions, in: Papers on General Topology and Applications, Ann. New York Acad. Sci., 767(1995), 28-38.
6. J.Dontchev, On pre-I-open sets and a decomposition of I-continuity, Banyan Math. J., 2(1996), 24-30.
7. E.Ekici, T.Noiri, $*$-extremally disconnected ideal topological spaces, Acta Math. Hungar., 122(2009), 81-90.
8. G.Gao, On topological spaces, Chinese Science Press, Beijing, 2008.
9. E.Hayashi, Topologies dened by local properties, Math. Ann., 156(1964), 205-215.
10. E.Hatir, T.Noiri, On β-I-open sets and a decomposition of almost I-continuity, Bull. Malays. Math. Sci. Soc., 29(2006), 119-124.
11. E.Hatir, T.Noiri, Weakly pre-I-open sets and decomposition of continuity, Acta Math. Hungar., 106(2005), 227-238.
12. E.Hatir, T.Noiri, On semi-I-open sets and semi-I-continuous functions, Acta Math. Hungar., 107(2005), 345-353.

13. D.S.Jankovic, T.R.Hamlett, New topologies from old via ideals, Amer. Math. Monthly, 97(1990), 295-310.
14. D.S.Jankovic, I.L.Reilly, On semiseparation properties, Indian J. Pure Appl. Math., 16(1985), 957-964.
15. V.Jeyanthi, V.Renukadevi, D.Sivaraj, Weakly I-continuous functions, Acta Math. Hungar., 113(2006), 319-324.
16. K.Kuratowski, Topology, Academic Press, New York, 1966.
17. E.D.Khalimsky, R.Kopperman, P.R.Meyer, Computer graphics and connected topologies on finite ordered sets, Topology Appl., 36(1990), 1-17.
18. T.Y.Kong, R.Kopperman, P.R.Meyer, A topological approach to digital topology, Amer. Math. Monthly, 98(1991), 901-917.
19. N.Levine, A decomposition of continuity in topological spaces, Amer. Math. Monthly, 68(1961), 44-46.
20. N.Levine, Semi-open sets and semi-continuity in topological spaces, Amer. Math. Monthly, 70(1963), 36-41.
21. W.K.Min, Weak continuity on generalized topological spaces, Acta Math. Hungar., 124(2009), 73-81.
22. W.K.Min, Almost continuity functions on generalized topological spaces, Acta Math. Hungar., 125(2009), 121-125.
23. A.S.Mashhour, M.E.Abd El-Monsef, S.N.El-Deep, On precontinuous and weak precontinuous mappings, Proc. Math. Phys. Soc. Egypt, 53(1982), 47-53.
24. R.L.Newcomb, Topologies which are compact modulo an ideal, Ph.D.Dissertations, University of California at Santa Barbara, 1967.
25. A.A.Nasef, T.Noiri, Some weak forms of almost continuity, Acta Math. Hungar., 74(1997), 211-219.
26. V.Renukadevi, D.Sivaraj, T.Tamizhchelvam, Codense and completely codense ideals, Acta Math. Hungar., 108(2005), 197-205.
27. M.H.Stone, Applications of the theory of Boolean rings to general topology, Trans. Amer. Math. Soc., 41(1937), 375-381.
28. M.Singal, A.Singal, Almost continuous mappings, Yokohama Math. J., 16(1968), 63-73.
29. J.Tong, Expansion of open sets and decomposition of continuous mappings, Rend. Circ. Mat. Palermo, 43(1994), 303-308.
30. R.Vaidyanathaswamy, The localisation theory in set topology, Proc. Indian Acad. Sci., 20(194), 51-61.
31. S.Yuksel, A.Acikgoz, T.Noiri, On δ-I-continuous functions, Turk J. Math., 29(2005), 39-51.

Received 15 01 2015, revised 25 06 2015

Shijiazhuang Institute of Technology,
Shijiazhuang, Hebei 050228,
P.R.China
E-mail address: `lixiaomin8846@126.com`