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Some remarks, generalizations and misprints in the integrals in Gradshteyn and Ryzhik

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Abstract. This paper presents some remarks, generalizations and misprints to the classic table of integrals of Gradshteyn and Ryzhik. It is noted that the conditions for convergence in a number of integrals, that in the original source were obtained via Bessel transforms, are too restrictive. Also, a property of the parabolic cylinder function is used to simplify one of the integrals in Gradshteyn and Ryzhik into a result that enables the calculation of integrals for squares and higher powers of the error function. Subsequently, an indefinite integral for the normal distribution is specialized into several expressions that are reported in Gradshteyn and Ryzhik. The paper ends by listing integrals for which the aforementioned overly restrictive conditions emerge and/or for which generalizations or misprints are found.

1. Introduction

This paper presents some remarks, generalizations and misprints to the classic table of integrals in [**12**]. Section 2 illustrates that the conditions for convergence in a number of integrals in [**12**] are too restrictive. This observation finds its origin in the fact that [**12**] intensively used expressions that were obtained via Bessel transforms in [**8**]. For instance, in the case of the Hankel transform with kernel $J_\nu(xy)$ with the integral being evaluated with respect to x, [**8**] conditions the parameter y to be a positive real variable. The resulting integrals that appear in [**12**] often retain the latter condition, notwithstanding the fact that the solutions in a number of cases also hold for negative values of y. Section 3 deals with the integral **7.751.3** of which the solution consists of a sum of two products of two parabolic cylinder functions. We generalize and simplify **7.751.3** and show that **7.751.2** is a limiting case. Furthermore, it is shown that the simplified expression for **7.751.3** can be used to obtain integrals for squares and higher powers of the (complementary) error function. In a next step, the paper draws attention to the integrals for the error function and the normal distribution in [**17**], [**9**] and [**19**] that correct a number of entries in [**12**] and

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that also can be used to extend results in [**12**]. As an example, Section 4 shows that one of the indefinite integrals in [**19**] can be specialized into several definite integrals that are reported in [**12**]. Section 5 lists a number of integrals for which the aforementioned overly restrictive restrictions can be found, for which generalizations can be offered and/or in which misprints are detected.

2. A remark on the conditions for convergence in [12]

The conditions for convergence in [**12**] can be too restrictive, especially when the integral in the original source was obtained via Bessel transforms. We will illustrate this for some examples and list additional cases in Section 5.

In [**8**], the Hankel transform of order ν is defined in terms of the kernel $J_\nu(xy)$ with y being a positive real variable (see p. 3 in [**8**]). For instance, (16) on p. 19 in [**8**] specifies the Hankel transform of order 1, $g(y;1)$, for $f(x) = x^{-\frac{1}{2}} \sin\left(\frac{1}{4}ax^2\right)$ with $a > 0$ as $g(y;1) = \int_0^\infty f(x) J_1(xy)(xy)^{\frac{1}{2}} dx = y^{-\frac{1}{2}} \sin\left(\frac{y^2}{a}\right)$ with $y > 0$. This result is reproduced in **6.686.5** as

$$\int_0^\infty \sin\left(\frac{1}{4}ax^2\right) J_1(xy)\, dx = \frac{1}{y} \sin\left(\frac{y^2}{a}\right), \ a > 0, \ y > 0,$$

where $J_1(z)$ denotes the Bessel function of the first kind with order 1. However, the two conditions for convergence in **6.686.5** are too restrictive. In fact, the solution also holds for negative values of y as can be inferred from the series expansion for $J_1(z)$ in **8.441.2**

$$J_1(z) = \frac{z}{2} \sum_{k=0}^\infty \frac{(-1)^k z^{2k}}{2^{2k} k! (k+1)!}.$$

Also, a may take on negative values given

$$\sin(z) = -\sin(-z).$$

The condition for convergence in **6.686.5** thus is less restrictive at $a \neq 0, \ y \neq 0$.

A second example evolves around the Y-transform in [**8**] for which the kernel is the Bessel function of the second kind, $Y_\nu(xy)$, again for $y > 0$ (see p. 93 in [**8**]). For example, the result in (1) on p. 105 of [**8**] is reproduced in [**12**] as **6.611.2**

$$\int_0^\infty e^{-\alpha x} Y_\nu(bx)\, dx = \left(\alpha^2 + b^2\right)^{-\frac{1}{2}} \operatorname{cosec}(\nu\pi) \times \left\{ b^\nu \left[\left(\alpha^2 + b^2\right)^{\frac{1}{2}} + \alpha\right]^{-\nu} \cos(\nu\pi) \right.$$

$$\left. -b^{-\nu} \left[\left(\alpha^2 + b^2\right)^{\frac{1}{2}} + \alpha\right]^{\nu} \right\}, \ \operatorname{Re}\alpha > 0, \ b > 0, \ |\operatorname{Re}\nu| < 1.$$

However, negative values for b must not prevent convergence as can be verified by numerical integration. Actually, the condition for convergence is $\operatorname{Re}(a \pm ib) > 0, \ |\operatorname{Re}\nu| <$

1. This is confirmed by noting that **6.611.2** can be obtained as a limiting case of **6.621.2**

$$\int_0^\infty e^{-\alpha x} Y_\nu(bx)\, x^{\mu-1} dx =$$

$$\cot(\nu\pi)\frac{\left(\frac{b}{2}\right)^\nu \Gamma(\nu+\mu)}{\sqrt{(\alpha^2+b^2)^{\nu+\mu}}\Gamma(\nu+1)} F\left(\frac{\nu+\mu}{2}, \frac{\nu-\mu+1}{2}; \nu+1; \frac{b^2}{\alpha^2+b^2}\right)$$

$$-\operatorname{cosec}(\nu\pi)\frac{\left(\frac{b}{2}\right)^{-\nu} \Gamma(\mu-\nu)}{\sqrt{(\alpha^2+b^2)^{\mu-\nu}}\Gamma(1-\nu)} F\left(\frac{\mu-\nu}{2}, \frac{1-\nu-\mu}{2}; 1-\nu; \frac{b^2}{\alpha^2+b^2}\right),$$

$$\operatorname{Re}\mu \geqslant |\operatorname{Re}\nu|, \ \operatorname{Re}(\alpha \pm ib) > 0,$$

where $F(a, b; c; z)$ is the Gauss hypergeometric function. Using $\mu = 1$ and applying the identity

$$F\left(\frac{1+q}{2}, \frac{q}{2}; 1+q; z\right) = 2^q \left(1+\sqrt{1-z}\right)^{-q}$$

indeed reduces **6.621.2** into **6.611.2**. The condition $\operatorname{Re}\mu \geqslant |\operatorname{Re}\nu|, \ \operatorname{Re}(\alpha \pm ib) > 0$ of **6.621.2** accordingly simplifies into $\operatorname{Re}(a \pm ib) > 0, \ |\operatorname{Re}\nu| < 1$.

We end this section with an example in [**12**] for which the condition $y > 0$ from [**8**] is required, but for which the solutions under alternative conditions can easily be found. For example, **6.566.5** is specified as

$$\int_0^\infty x^{-\nu} J_\nu(ax)\frac{dx}{x^2+b^2} = \frac{\pi}{2b^{\nu+1}}\left[I_\nu(ab) - \boldsymbol{L}_\nu(ab)\right], \ a > 0, \operatorname{Re} b > 0, \ \operatorname{Re}\nu > -\frac{5}{2}$$

and is based on (11) on p. 426 of [**21**] and (14) on p. 23 in [**8**]. However, solutions for other combinations in a and b can be calculated from the above result and the definitions in **8.476**, **8.550** and **8.553** as

$$\begin{aligned}\int_0^\infty x^{-\nu} J_\nu(ax)\frac{dx}{x^2+b^2} &= \frac{-\pi}{2(-b)^{\nu+1}}\left[I_\nu(-ab) - \boldsymbol{L}_\nu(-ab)\right], \ a<0, \ \operatorname{Re} b > 0 \ \operatorname{Re}\nu > -\frac{5}{2}, \\ &= \frac{\pi}{2(-b)^{\nu+1}}\left[I_\nu(-ab) - \boldsymbol{L}_\nu(-ab)\right], \ a>0, \operatorname{Re} b < 0, \ \operatorname{Re}\nu > -\frac{5}{2}, \\ &= \frac{\pi}{2(-b)^{\nu+1}}\left[I_\nu(-ab) + \boldsymbol{L}_\nu(-ab)\right], a<0, \ \operatorname{Re} b < 0, \ \operatorname{Re}\nu > -\frac{5}{2}.\end{aligned}$$

3. Simplifying an integral for the parabolic cylinder function

7.751.3 is specified as

$$\int_0^\infty J_0(xy)\, D_\nu(x)\, D_{\nu+1}(x)\, dx = \frac{1}{2y}\left[D_\nu(-y)\, D_{\nu+1}(y) - D_{\nu+1}(-y)\, D_\nu(y)\right],$$

where $D_\nu(z)$ denotes the parabolic cylinder function of order ν (see [**6**]). This result first can be generalized into

$$\int_0^\infty J_0(xy)\, D_\nu(ax)\, D_{\nu+1}(ax)\, dx = \frac{1}{2y}\left[D_\nu\left(-\frac{y}{a}\right) D_{\nu+1}\left(\frac{y}{a}\right) - D_{\nu+1}\left(-\frac{y}{a}\right) D_\nu\left(\frac{y}{a}\right)\right], \tag{3.1}$$

$y \neq 0,\ a > 0$ or $y \neq 0,\ a \neq 0,\ \nu = 0, 1, 2, ...$

The solution in (3.1) then can be written more compactly via a property of the parabolic cylinder function that [**20**] used in the derivation of the inverse Laplace transform for products of two parabolic cylinder functions. This simplification requires combining the derivative and the Wronskian of the parabolic cylinder function. The derivatives of $D_\nu(z)$ to the argument are given by

$$D_\nu'(z) = \frac{1}{2} z D_\nu(z) - D_{\nu+1}(z), \tag{3.2}$$

$$D_\nu'(-z) = \frac{1}{2} z D_\nu(-z) + D_{\nu+1}(-z), \tag{3.3}$$

which follow from (16) on p. 119 in [**6**]. The Wronskian is specified in (10) on p. 117 in [**6**] as

$$D_\nu(z)\, D_\nu'(-z) - D_\nu(-z)\, D_\nu'(z) = \frac{\sqrt{2\pi}}{\Gamma(-\nu)}. \tag{3.4}$$

Connecting (3.2)-(3.4) gives

$$D_\nu\left(\frac{z}{a}\right) D_{\nu+1}\left(-\frac{z}{a}\right) + D_\nu\left(-\frac{z}{a}\right) D_{\nu+1}\left(\frac{z}{a}\right) = \frac{\sqrt{2\pi}}{\Gamma(-\nu)},$$

through which (3.1) can be reduced into

$$\int_0^\infty J_0(xy)\, D_\nu(ax)\, D_{\nu+1}(ax)\, dx = \frac{1}{y}\left[\frac{\sqrt{\pi}}{\sqrt{2}\Gamma[-\nu]} - D_\nu\left(\frac{y}{a}\right) D_{\nu+1}\left(-\frac{y}{a}\right)\right],$$

$y \neq 0,\ a > 0$ or $y \neq 0,\ a \neq 0,\ \nu = 0, 1, 2, ...$

This result can be simplified further by noting that

$$\lim_{\nu \to n}\left[\frac{1}{\Gamma[-\nu]}\right] = 0, \text{ for } n = 0, 1, 2, ...,$$

see (6.1.7) in [**1**]. As a result, **7.751.3** can be rewritten as

(3.5)

$$\int_0^\infty J_0(xy)\, D_\nu(ax)\, D_{\nu+1}(ax)\, dx = \frac{1}{y}\left[\frac{\sqrt{\pi}}{\sqrt{2}\Gamma[-\nu]} - D_\nu\left(\frac{y}{a}\right) D_{\nu+1}\left(-\frac{y}{a}\right)\right],\ y \neq 0,\ a > 0,$$

(3.6)

$$= -\frac{1}{y} D_n\left(\frac{y}{a}\right) D_{n+1}\left(-\frac{y}{a}\right),\ y \neq 0,\ a \neq 0,\ n = 0, 1, 2, ...,$$

This result has interesting limiting cases. Setting ν at -1 in (3.5) allows to switch to the error function in view of the relations **9.253** and **9.254.1**, respectively

$$D_0(z) = \exp\left(-\frac{z^2}{4}\right),$$

$$D_{-1}(z) = \sqrt{\frac{\pi}{2}} \exp\left(\frac{z^2}{4}\right)\left(1 - \operatorname{erf}\left(\frac{z}{\sqrt{2}}\right)\right),$$

where $\operatorname{erf}(z)$ denotes the error function that is defined in **8.250.1** as

$$\operatorname{erf}(z) = \frac{2}{\sqrt{\pi}} \int_0^z e^{-t^2} dt. \tag{3.7}$$

Plugging the latter expressions into (3.5) and rescaling a gives

$$\int_0^\infty J_0(xy)\,(1 - \operatorname{erf}(ax))\, dx = \frac{1}{y} \operatorname{erf}\left(\frac{y}{2a}\right),\ y \neq 0,\ a \geqslant 0, \tag{3.8}$$

$$= \frac{1}{y}\left(2 + \operatorname{erf}\left(\frac{y}{2a}\right)\right),\ y > 0,\ a < 0, \tag{3.9}$$

$$= \frac{1}{y}\left(\operatorname{erf}\left(\frac{y}{2a}\right) - 2\right),\ y < 0,\ a < 0, \tag{3.10}$$

in which $1 - \operatorname{erf}(z)$ denotes the complementary error function (see **8.250.4**). The expression in (3.8) is directly obtained from (3.5), whereas the solutions in (3.9) and (3.10) are easily derived from (3.8) by using the following relations

$$\operatorname{erf}(x) = -\operatorname{erf}(-x),$$

$$J_0(xy) = J_0(-xy),$$

$$\int_0^\infty J_0(xy)\, dx = \frac{1}{|y|},$$

that follow from (3.7), the series expression for $J_0(z)$ in **8.441.1** and **6.511.1**, respectively. Note that the result in (3.8) also can be obtained from a result in [**12**], namely by evaluating a limiting case of **6.784.2**

$$\int_0^\infty x^\nu (1 - \Phi(ax))\, J_\nu(bx)\, dx = \sqrt{\frac{2}{\pi}} \frac{a^{\frac{1}{2}-\nu}\Gamma\left(\nu + \frac{1}{2}\right)}{b^{\frac{3}{2}}\Gamma\left(\nu + \frac{3}{2}\right)} \exp\left(-\frac{b^2}{8a^2}\right) M_{\frac{1}{2}\nu-\frac{1}{4}, \frac{1}{2}\nu+\frac{1}{4}} \exp\left(\frac{b^2}{4a^2}\right),$$

in which $\Phi(z)$ denotes the error function and $M_{\mu,\lambda}(z)$ is a Whittaker function. For $\nu=0$ and $b=y$, the Whittaker function can be rewritten as

$$M_{-\frac{1}{4},\frac{1}{4}}\left(\frac{b^2}{4a^2}\right)=\exp\left(-\frac{b^2}{8a^2}\right)\left(\frac{b^2}{4a^2}\right)^{\frac{3}{4}}\Phi\left(1,\frac{3}{2};\frac{b^2}{4a^2}\right),$$

where $\Phi(\alpha,\gamma;z)$ is a confluent hypergeometric function for which we used the relation $M_{\kappa,\mu}(z)=\exp(-z/2)\,z^{\mu+1/2}\Phi(1/2-\kappa+\mu,2\mu+1;z)$ in (1) on p. 264 of [**5**]. The confluent hypergeometric function and the error function in turn are connected by **8.253.1**

$$\operatorname{erf}(z)=\frac{2}{\sqrt{\pi}}z\exp\left(-z^2\right)\Phi\left(1,\frac{3}{2};z^2\right),$$

such that we obtain

$$M_{-\frac{1}{4},\frac{1}{4}}\left(\frac{b^2}{4a^2}\right)=\frac{1}{2}\sqrt{\pi}\exp\left(\frac{b^2}{8a^2}\right)\left(\frac{b^2}{4a^2}\right)^{\frac{1}{4}}\operatorname{erf}\left(\frac{2a}{y}\right).$$

Using the latter result then confirms that **6.784.2** indeed simplifies into (3.8).

The integral (3.5) can also be used to obtain solutions for integrands that contain the square of a complementary error function. Setting ν at -2 in (3.5) and using **9.254.2**

$$D_{-2}(z)=\exp\left(-\frac{z^2}{4}\right)-z\sqrt{\frac{\pi}{2}}\exp\left(\frac{z^2}{4}\right)\left(1-\operatorname{erf}\left(\frac{z}{\sqrt{2}}\right)\right),$$

which yields

$$\int_0^\infty J_0(xy)\,x\exp\left(a^2x^2\right)[1-\operatorname{erf}(ax)]^2\,dx=$$

$$\frac{2}{ay\sqrt{\pi}}\operatorname{erf}\left(\frac{y}{2a}\right)+\frac{1}{2a^2}\exp\left(\frac{y^2}{4a^2}\right)\left(\left[\operatorname{erf}\left(\frac{y}{2a}\right)\right]^2-1\right),\ y\neq 0,\ a>0.$$

Setting ν at larger negative integer values then offers expressions for integrands with complementary error functions that are raised to third and higher powers.

The rewritten expression for **7.751.3** also has **7.751.2** as a limiting case. Hereto, we use (3.6) and the linear relation between parabolic cylinder functions in **9.248.1**

$$D_p(z)=e^{p\pi i}D_p(-z)+\frac{\sqrt{2\pi}}{\Gamma(-p)}e^{\pi(p+1)i/2}D_{-p-1}(-iz),$$

which for non-negative integers gives

$$D_n(-z)=(-1)^n D_n(z),\ \text{for } n=0,1,2,...$$

Plugging the latter result into (3.6) then gives **7.751.2**

$$\int_0^\infty J_0(xy)\,D_n(ax)\,D_{n+1}(ax)\,dx=\frac{(-1)^n}{y}D_n\left(\frac{y}{a}\right)D_{n+1}\left(\frac{y}{a}\right),\ y\neq 0,\ a\neq 0,\ n=0,1,2,....$$

4. An indefinite integral for the error function and its specializations in [12]

The tables in [**17**], [**9**] and [**19**] offer a wide variety of definite and indefinite integrals for the error function and the normal distribution that confirm, extend and also correct integrals in [**12**]. We will intensively refer to [**17**] and [**9**] in Section 5 when addressing a number of misprints and errata. This section employs an indefinite integral from [**19**] and shows that several of its specializations are reported in [**12**].

The following indefinite integral can be found in (10,010.3) in [**19**]

$$\begin{aligned}\int G'(y)\,G(\alpha+\beta y)\,dy &= T\left(y,\frac{\alpha}{y\sqrt{1+\beta^2}}\right)+T\left(\frac{\alpha}{\sqrt{1+\beta^2}},\frac{y\sqrt{1+\beta^2}}{\alpha}\right)-T\left(y,\frac{\alpha+\beta y}{y}\right)\\ &-T\left(\frac{\alpha}{\sqrt{1+\beta^2}},\frac{\alpha\beta+y\left(1+\beta^2\right)}{\alpha}\right)+G(y)\,G\left(\frac{\alpha}{\sqrt{1+\beta^2}}\right),\ \alpha \text{ real},\ \beta \text{ real}\end{aligned} \tag{4.1}$$

in which $G(z)$ and $T(h,z)$ denote the cumulative standard normal distribution and Owen's T function [**18**], respectively. The latter two functions, see [**19**], are defined as

$$G(z)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{z}\exp\left(-\frac{1}{2}t^2\right)dt,$$

$$T(h,z)=\frac{1}{2\pi}\int_{0}^{z}\exp\left(-\frac{1}{2}h^2\left(1+t^2\right)\right)\frac{dt}{1+t^2}.$$

Note that Owen's T function is implemented in *Mathematica* as OwenT[h,a]. Intensive use will be made of the following properties of Owen's T function (see [**18**])

$$\begin{aligned}T(h,z)&=-T(h,-z),\\ T(h,z)&=T(-h,z),\\ T(h,0)&=0,\\ T(0,z)&=\frac{1}{2\pi}\arctan z,\\ T(h,1)&=\frac{1}{2}G(h)\left(1-G(h)\right),\\ T(h,\infty)&=\begin{cases}\frac{1}{2}\left(1-G(h)\right) & \text{, for } h\geqslant 0,\\ \frac{1}{2}G(h) & \text{, for } h\leqslant 0.\end{cases}\end{aligned} \tag{4.2}$$

As the results in [**12**] are expressed in terms of the error function, we rewrite the above expressions via the connection between the error function and $G(z)$ in (26.2.29) in [**1**]

$$G(z)=\frac{1}{2}\operatorname{erf}\left(\frac{x}{\sqrt{2}}\right)+\frac{1}{2}.$$

The last two properties of the T function in (4.2) then can be rewritten as

$$T(h,1)=\frac{1}{8}\left(1-\left[\operatorname{erf}\left(\frac{h}{\sqrt{2}}\right)\right]^2\right),$$

$$T(h,\infty)=\begin{cases}\frac{1}{4}\left(1-\operatorname{erf}\left(\frac{h}{\sqrt{2}}\right)\right) & \text{, for } h\geqslant 0,\\ \frac{1}{4}\left(1+\operatorname{erf}\left(\frac{h}{\sqrt{2}}\right)\right) & \text{, for } h\leqslant 0.\end{cases}$$

In what follows, we also frequently employ the following properties of the error function

$$\begin{aligned}&\operatorname{erf}(+\infty)=1,\\ &\operatorname{erf}(-\infty)=-1,\\ &\operatorname{erf}(0)=0,\end{aligned}\tag{4.3}$$

that directly follow from the definition of the error function in (3.7) and from (a slightly generalized) **3.321.3**

$$\int_0^\infty e^{-q^2x^2}\,dx=\frac{\sqrt{\pi}}{2\sqrt{q^2}},\ \operatorname{Re}q^2>0.$$

Expressing (4.1) in terms of the error function yields:

$$\begin{aligned}\int e^{-px^2}\operatorname{erf}(a+bx)\,dx \;=\;& \frac{2\sqrt{\pi}}{\sqrt{p}}\left\{T\left(x\sqrt{2p},\frac{a}{x\sqrt{p+b^2}}\right)+T\left(\frac{a\sqrt{2p}}{\sqrt{p+b^2}},\frac{x\sqrt{p+b^2}}{a}\right)\right.\\ &-T\left(x\sqrt{2p},\frac{a+bx}{x\sqrt{p}}\right)-T\left(\frac{a\sqrt{2p}}{\sqrt{p+b^2}},\frac{ab+x\left(p+b^2\right)}{a\sqrt{p}}\right)\\ &\left.+\frac{1}{4}\operatorname{erf}\left(\frac{a\sqrt{p}}{\sqrt{p+b^2}}\right)\left(1+\operatorname{erf}\left(x\sqrt{p}\right)\right)+\frac{1}{4}\right\}\end{aligned}\tag{4.4}$$

$\operatorname{Re}p>0$, a real, b real

The result in (4.4) can be used to compute several definite integrals in [**12**] such as **8.259.1**, **8.250.7**, **8.250.9** and, in terms of the complementary error function, **6.285.1**. These specializations will be illustrated here and also point to two misprints and a generalization in one of the results in [**12**].

First, **8.259.1** is given by

$$\int_{-\infty}^{+\infty} e^{-px^2}\operatorname{erf}(a+bx)\,dx=\frac{\sqrt{\pi}}{\sqrt{p}}\operatorname{erf}\left(\frac{a\sqrt{p}}{\sqrt{p+b^2}}\right),\ \operatorname{Re}p>0,\ a \text{ real},\ b \text{ real}$$

Taking $\operatorname{Re}p>0$, assuming first $a>0$ in the evaluation of the limits in (4.4) and using the properties in (4.2) and (4.3) straightforwardly yields the above solution. It is easy to show that the solution in **8.259.1** also arises for $a=0$ and $a<0$.

Second, **8.250.7** is given by

$$\int_0^p e^{-x^2} \operatorname{erf}(p-x)\, dx = \frac{\sqrt{\pi}}{2}\left[\operatorname{erf}\left(\frac{p}{\sqrt{2}}\right)\right]^2.$$

Obtaining the latter expression as a limiting case of (4.4) is somewhat more involved. Using $p = 1$, $a = p$ and $b = -1$ in (4.4) and employing the properties in (4.2) and (4.3) gives

$$\int_0^p e^{-x^2} \operatorname{erf}(p-x)\, dx =$$

$$\frac{\sqrt{\pi}}{2}\left[\operatorname{erf}\left(\frac{p}{\sqrt{2}}\right)\right]^2 + 2\sqrt{\pi}\left\{T\left(p\sqrt{2}, \frac{1}{\sqrt{2}}\right) + T\left(p, \sqrt{2}\right) - \frac{1}{4} + \frac{1}{4}\operatorname{erf}\left(\frac{p}{\sqrt{2}}\right)\operatorname{erf}(p)\right\}.$$

The second term in this solution vanishes in view of the identity in (3.5) in [**18**]

$$T(h,a) = \tfrac{1}{2}G(h) + \tfrac{1}{2}G(ah) - G(h)\,G(ah) - T\left(ah, \frac{1}{a}\right), a \geqslant 0,$$

which in terms of the error function is

$$T(h,a) = \frac{1}{4} - \frac{1}{4}\operatorname{erf}\left(\frac{h}{\sqrt{2}}\right)\operatorname{erf}\left(\frac{ah}{\sqrt{2}}\right) - T\left(ah, \frac{1}{a}\right), a \geqslant 0.$$

Using the latter result indeed gives **8.250.7**. Integration by parts of (4.4) and the above simplifications also straightforwardly give **8.250.8**, for which, however, is to be noted that the term $\Phi\left(-\frac{x^2}{2}\right)$ in the solution in [**12**] is to be replaced by $\exp\left(-\frac{p^2}{2}\right)$.

Obtaining **8.250.9** evolves along similar lines such that we refrain from presenting details. However, it should be noted that the solution in **8.250.9** contains a misprint as (4.4) can be shown to simplify into $-\sqrt{\pi}\Phi(a)\,\Phi(b)$.

A final specialization refers to **6.285.1**

$$\int_0^\infty (1-\operatorname{erf}(x))\, e^{-\mu^2 x^2} dx = \frac{\arctan \mu}{\sqrt{\pi}\mu}, \operatorname{Re}\mu > 0.$$

Rewriting (4.4) towards the complementary error function and evaluating the limits gives

$$\int_0^\infty (1-\operatorname{erf}(x))\, e^{-\mu^2 x^2} dx =$$

$$\frac{\sqrt{\pi}}{2\sqrt{\mu^2}} - \frac{1}{\sqrt{\pi}\sqrt{\mu^2}}\left\{\arctan\frac{2}{\sqrt{\mu^2}} + \arctan\frac{2+\mu^2}{\sqrt{\mu^2}} - \arctan\frac{1}{\sqrt{1+\mu^2}} - \arctan\sqrt{1+\mu^2}\right\}$$

The term within parentheses then can be simplified via **1.627.1**

$$\arctan x + \arctan\frac{1}{x} = \frac{\pi}{2}$$

and the identity

$$\arctan\frac{2}{x}+\arctan\frac{2+x^2}{x}=\pi-\arctan x.$$

This yields

$$\int_0^\infty (1-\operatorname{erf}(x))\, e^{-\mu^2x^2}dx=\frac{\arctan\sqrt{\mu^2}}{\sqrt{\pi}\sqrt{\mu^2}},\operatorname{Re}\mu^2>0,$$

which generalizes **6.285.1** to values of μ with negative real part.

5. Remarks, generalizations and misprints in [12]

2.33.16: for $\exp\left(\sqrt{\beta}x\right)$, read $\operatorname{erf}\left(\sqrt{\beta}x\right)$.

3.311.1: $\operatorname{Re}p>0$ is to be included. This integral is derived via the method of brackets in [**11**].

3.311.5: for $\operatorname{Re}\nu<1$, read $\operatorname{Re}\nu<0$.

3.312.1: for $\operatorname{Re}\beta>0,\ \operatorname{Re}\nu>0,\ \operatorname{Re}\mu>0$, read $\operatorname{Re}\beta>0,\ \operatorname{Re}\nu>1,\ \operatorname{Re}\mu>0$.

3.318.2: for $\sqrt{\pi e}^{-\frac{u}{2}(\mu+\nu)}$, read $\sqrt{\pi}e^{-\frac{u}{2}(\mu+\nu)}$. This misprint is reported in [**13**].

3.321.3: for $\dfrac{\sqrt{\pi}}{2q}$, $q>0$, read $\dfrac{\sqrt{\pi}}{2\sqrt{q^2}}$, $\operatorname{Re}q^2>0$.

3.322.1: delete $\operatorname{Re}\beta>0,\ u>0$.

3.323.2: for $\dfrac{\sqrt{\pi}}{p}$, read $\dfrac{\sqrt{\pi}}{\sqrt{p^2}}$.

3.323.3: add $\operatorname{Re}a>0$.

3.323.4: add $\operatorname{Re}\beta^2>0,\ \operatorname{Re}\gamma^2>0$.

3.354.5: for $\dfrac{\pi}{a}$, read $\dfrac{\pi}{|a|}$ as noted in [**13**].

3.416.3: for 2^{2^n}, read 2^{2n} as reported in [**13**].

3.417.1: for $\dfrac{\pi}{2ab}\ln\left(\dfrac{b}{a}\right)$, $ab>0$, read $\dfrac{\pi}{2\,|ab|}\ln\left|\dfrac{b}{a}\right|$, $a\neq0,\ b\neq0$.

3.462.25: for $\operatorname{Re}a>0,\ \operatorname{Re}b>0$, read $\operatorname{Re}a>0,\ \operatorname{Re}p>0$. This integral is derived in [**10**].

3.466.1: the solution for negative real part in b is $-\left[1+\Phi\left(b\mu\right)\right]\dfrac{\pi}{2b}e^{b^2\mu^2}$ $\operatorname{Re}b<0, |\arg\mu|<\frac{\pi}{4}$.

3.691.2: for $S\left(\sqrt{a}\right)$, read $S\left(\sqrt{\dfrac{2a}{\pi}}\right)$.

3.691.3: for $C\left(\sqrt{a}\right)$, read $C\left(\sqrt{\dfrac{2a}{\pi}}\right)$.

3.691.4: for $C\left(\dfrac{b}{\sqrt{a}}\right)$, read $C\left(b\sqrt{\dfrac{2}{a\pi}}\right)$; for $S\left(\dfrac{b}{\sqrt{a}}\right)$, read $S\left(b\sqrt{\dfrac{2}{a\pi}}\right)$.

3.691.6: for $C\left(\frac{b}{\sqrt{a}}\right)$, read $C\left(b\sqrt{\frac{2}{a\pi}}\right)$; for $S\left(\frac{b}{\sqrt{a}}\right)$, read $S\left(b\sqrt{\frac{2}{a\pi}}\right)$.

3.691.8: for $C\left(\frac{a}{2b}\right)$, read $C\left(\frac{a}{b\sqrt{2\pi}}\right)$; for $S\left(\frac{a}{2b}\right)$, read $S\left(\frac{a}{b\sqrt{2\pi}}\right)$.

3.691.9: for $C\left(\frac{a}{2b}\right)$, read $C\left(\frac{a}{b\sqrt{2\pi}}\right)$; for $S\left(\frac{a}{2b}\right)$, read $S\left(\frac{a}{b\sqrt{2\pi}}\right)$.

3.725.3: the solution must depend on β (as also noted in [**4**]).
Let $\gamma_1 = \frac{\pi}{2\beta^2}e^{-\beta b}\sinh(a\beta)$, $\gamma_2 = \frac{-\pi}{2\beta^2}e^{\beta b}\sinh(a\beta)$, $\gamma_3 = \frac{-\pi}{2\beta^2}e^{-a\beta}\cosh(b\beta) + \frac{\pi}{2\beta^2}$ and $\gamma_4 = \frac{-\pi}{2\beta^2}e^{a\beta}\cosh(b\beta) + \frac{\pi}{2\beta^2}$. The solution for $0 < a < b,\ \mathrm{Re}\,\beta > 0$ and $a < 0 < b,\ \mathrm{Re}\,\beta > 0$ is γ_1 and the solution for $b < a < 0,\ \mathrm{Re}\,\beta < 0$ is $-\gamma_1$. The solution for $0 < a < b,\ \mathrm{Re}\,\beta < 0$ and $a < 0 < b,\ \mathrm{Re}\,\beta < 0$ is γ_2 and the solution for $b < a < 0,\ \mathrm{Re}\,\beta > 0$ is $-\gamma_2$. The solution for $0 < b < a,\ \mathrm{Re}\,\beta > 0$ and $b < 0 < a,\ \mathrm{Re}\,\beta > 0$ is γ_3 and the solution for $a < b < 0,\ \mathrm{Re}\,\beta < 0$ is $-\gamma_3$.
The solution for $0 < b < a,\ \mathrm{Re}\,\beta < 0$ and $b < 0 < a,\ \mathrm{Re}\,\beta < 0$ is γ_4 and the solution for $a < b < 0,\ \mathrm{Re}\,\beta > 0$ is $-\gamma_4$.

3.755.1: for $a > 0$, read $a > 0,\ \mathrm{Re}\,b > 0$.

3.772.5: for ET I 12(4), read ET I 12(14).

4.358.2: for $\zeta(2, \nu - 1)$, read $\zeta(2, \nu)$ as noted in [**22**] and [**15**] in which also a derivation of this integral can be found.

6.282.2: add $\mathrm{Re}\,\mu > 0$.

6.283.1: for $\mathrm{Re}\,\alpha > 0,\ \mathrm{Re}\,\beta > \mathrm{Re}\,\alpha$, read $\mathrm{Re}\,\beta < 0,\ \mathrm{Re}\,\alpha > \mathrm{Re}\,\beta$.

6.285.1 replace μ in the solution and condition by $\sqrt{\mu^2}$.

6.285.2: for $-\frac{1}{2ai\sqrt{\pi}}$, read $\frac{1}{2ai\sqrt{\pi}}$.

6.291: for $\frac{\mu}{a}$, read $\frac{\mu}{4}$.

6.295.2: for $-\frac{1}{\mu^2}$, read $-\frac{1}{\mu}$ as can be seen from (31) on p. 9 of [**17**].

6.296: for $|\arg\mu| < \frac{\pi}{4},\ a > 0$, read $|\arg\mu| < \frac{\pi}{4},\ a$ real.

6.297.1: for $\mathrm{Re}\,\beta > 0,\ \mathrm{Re}\,\mu > 0$, read $\mathrm{Re}\,\beta > 0,\ \mathrm{Re}\,\mu > 0,\ \mathrm{Re}\left(\gamma^2 - \mu\right) < 0$.

6.297.2: for $a > 0,\ b > 0,\ \mathrm{Re}\,\mu > 0$, read $b > 0,\ \mathrm{Re}\left(\mu^2 - a^2\right) > 0$.

6.297.3: for $a > 0,\ b > 0,\ \mathrm{Re}\,\mu > 0$, read $b > 0,\ \mathrm{Re}\,\mu > 0$.

6.298: for $a > 0,\ b > 0,\ \mathrm{Re}\,\mu > 0$, read $b > 0,\ \mathrm{Re}\,\mu > 0,\ \mathrm{Re}\left(\mu - a^2\right) > 0$.

6.299: for $K_\nu\left(a^2\right)$, read $K_\nu\left(\frac{1}{2}a^2\right)$ as noted in (36) on p. 152 of [**9**].

6.311: for $a > 0,\ b > 0$, read $a > 0,\ b \neq 0$. For negative a, the solution is
$\frac{1}{b}\left(1 + e^{-\frac{b^2}{4a^2}}\right)$, $a < 0,\ b \neq 0$.

6.312: for $a>0,\ b>0$, read $a>0,\ b>0,\ a<\sqrt{b}$. For $a>\sqrt{b}$, the solution is

$$\frac{1}{4\sqrt{2\pi b}}\left(\ln\frac{b+a^2+a\sqrt{2b}}{b+a^2-a\sqrt{2b}}+2\arctan\frac{a\sqrt{2b}}{b-a^2}+2\pi\right),\ a>0,\ b>0,\ a>\sqrt{b},$$

see (10) on p. 154 of [**9**].

6.314.1: the integral and its solution should be replaced by

$$\int_0^\infty \sin(bx)\,\Phi\left[\sqrt{\frac{a}{x}}\right]dx=b^{-1}\left(1-\exp\left[-(2ab)^{\frac{1}{2}}\right]\cos\left[(2ab)^{\frac{1}{2}}\right]\right),\ \operatorname{Re}a>0,\ b>0,$$

as noted in (16) on p. 155 in [**9**].

6.314.2: the integral and its solution should be replaced by

$$\int_0^\infty \cos(bx)\,\Phi\left[\sqrt{\frac{a}{x}}\right]dx=b^{-1}\exp\left[-(2ab)^{\frac{1}{2}}\right]\sin\left[(2ab)^{\frac{1}{2}}\right],\ \operatorname{Re}a>0,\ b>0,$$

see (17) on p. 155 in [**9**].

6.315.3: for $a>0,\ b>0$, read $a>0,\ b\neq 0$.

6.315.4: for $\operatorname{Ei}\left[\frac{p}{4a^2}\right]$, read $\operatorname{Ei}\left[-\frac{p}{4a^2}\right]$; for $a>0,\ b>0,\ p>0$, read $a>0,\ b>0,\ p\neq 0$.

6.315.5: for $a>0,\ b>0$, read $a>0,\ b>0,\ a<\sqrt{b}$. For $a>\sqrt{b}$, the solution is

$$\frac{1}{2\sqrt{2\pi b}}\left(\ln\frac{b+a\sqrt{2b}+a^2}{b-a\sqrt{2b}+a^2}+2\arctan\left[\frac{a\sqrt{2b}}{b-a^2}\right]+2\pi\right),\ a>0,\ b>0,\ a>\sqrt{b}.$$

6.317: for $b>0$, read $\operatorname{Re}a^2>0,\ b>0$. For negative b, the solution is $-\frac{i}{a}\frac{\sqrt{\pi}}{2}e^{-\frac{b^2}{4a^2}}$ $\operatorname{Re}a^2>0,\ b<0$.

6.318: the correct solution is

$$\frac{e^{-p^2}-1}{2p}+\frac{\sqrt{\pi}}{2}\Phi(p),\ \operatorname{Re}p>0.$$

6.511.6: the formula can be generalized by modifying the expression in ET II 7(2) into

$$\int_0^a J_0(xy)\,dx=aJ_0(ay)+\frac{\pi a}{2}\left[J_1(ay)\,\mathbf{H}_0(ay)-J_0(ay)\,\mathbf{H}_1(ay)\right],\ a>0.$$

6.511.7: the formula in ET II 18(1) is more general and the restriction on y can be relaxed

$$\int_0^a J_1(xy)\,dx=\frac{1}{y}\left(1-J_0(ay)\right),\ a>0, y\neq 0.$$

6.511.8: the formula can be generalized by modifying the expression in ET II 7(3) into

$$\int_a^\infty J_0(xy)\,dx = \frac{1}{|y|} - aJ_0(ay) + \frac{\pi a}{2}\left[J_0(ay)\,\mathbf{H}_1(ay) - J_1(ay)\,\mathbf{H}_0(ay)\right],\ y \neq 0.$$

6.511.9: the formula in ET II 18(2) is more general and the restriction on y can be relaxed

$$\int_a^\infty J_1(xy)\,dx = \frac{1}{y}J_0(ay),\ y \neq 0.$$

6.512.9: for $a > 0,\ b > 0$, read $a > 0,\ b \neq 0$.

6.512.10: for $a > 0,\ b > 0$, read $a > 0,\ b \neq 0,\ a > |b|$.

6.516.1: for negative b, the solution is $-b^{-1}J_\nu\left(\frac{a^2}{4b}\right),\ a > 0,\ b < 0,\ \operatorname{Re}\nu > -\frac{1}{2}$.

6.516.4: for $a > 0,\ b > 0$, read $a > 0,\ b > 0,\ \operatorname{Re}\nu > -\frac{1}{2}$.

6.521.2: for $\operatorname{Re}a > 0,\ b > 0,\ \operatorname{Re}\nu > -1$, read $\operatorname{Re}(a \pm ib) > 0,\ \operatorname{Re}\nu > -1$. In fact, **6.521.2** is the limiting case of **6.576.3** for $\lambda = -1,\ \mu = \nu$ with $F(q+1, 1; q+1, x) = \frac{1}{1-x}$. The condition $\operatorname{Re}(a \pm ib) > 0,\ \operatorname{Re}(\nu - \lambda + 1) > |\operatorname{Re}\mu|$ in **6.576.3** then simplifies into $\operatorname{Re}(a \pm ib) > 0,\ \operatorname{Re}\nu > -1$.

6.521.7: for $a > 0,\ b > 0$, read $a > 0$.

6.521.8: for $a > b > 0$, read $a > |b| \geqslant 0$.

6.521.9: for $a > b > 0$, read $a > |b| \geqslant 0$.

6.521.12: for $a > 0, b > 0$, read $a > 0$.

6.521.13: for $a > b > 0$, read $a > 0$.

6.521.14: for $a > b > 0$, read $a > |b| \geqslant 0$.

6.521.15: for $a > b > 0$, read $a > |b| \geqslant 0$.

6.522.4: for $\operatorname{Re}b > \operatorname{Re}a,\ c > 0$, read $\operatorname{Re}b > \operatorname{Re}a$; for $\operatorname{Re}b > \operatorname{Re}c,\ a > 0$, read $\operatorname{Re}b > \operatorname{Re}c$.

6.522.5: for $\operatorname{Re}b > |\operatorname{Im}a|,\ c > 0$, read $\operatorname{Re}b > |\operatorname{Im}a|$; for $\operatorname{Re}a > |\operatorname{Im}b|,\ c > 0$, read $[\operatorname{Re}a > |\operatorname{Im}b|$.

6.524.2: for $\frac{\pi}{8ab} - \frac{1}{4ab}\arcsin\left(\frac{b^2-a^2}{b^2+a^2}\right),\ a > 0,\ b > 0$, read $\frac{\pi}{8|a|b} - \frac{1}{4|a|b}\arcsin\left(\frac{b^2-a^2}{b^2+a^2}\right),\ a \neq 0,\ b > 0$.

6.525.1: for $c > 0,\ \operatorname{Re}b \geqslant |\operatorname{Im}a|,\ \operatorname{Re}a > 0$, read $\operatorname{Re}b > |\operatorname{Im}a|$; for $\operatorname{Re}a > |\operatorname{Im}b|,\ \operatorname{Re}b > 0,\ c > 0$, read $\operatorname{Re}a > |\operatorname{Im}b|$.

6.525.2: for $\operatorname{Re}b > |\operatorname{Re}a|,\ c > 0$, read $\operatorname{Re}b > |\operatorname{Re}a|$.

6.525.3: for $K_0(bx)$, read $K_1(bx)$; for $\operatorname{Re}a > |\operatorname{Im}b|,\ c > 0$, read $\operatorname{Re}b > 0$.

6.526.1: for $(2a)^{-1}J_{\frac{1}{2}\nu}\left(\frac{b^2}{4a}\right),\ a > 0,\ b > 0,\ \operatorname{Re}\nu > -1$, read $(2|a|)^{-1}J_{\frac{1}{2}\nu}\left(\frac{b^2}{4a}\right)$

$a \neq 0,\ b \geqslant 0, \quad \operatorname{Re}\nu > -1.$

6.532.4: for $a > 0,\ \operatorname{Re} k > 0$, read $a > 0$ and $\operatorname{Re} k > 0$ or $a < 0$ and $\operatorname{Re} k < 0$. The solution is $K_0(-ak)$ for $a > 0$ and $\operatorname{Re} k < 0$ or $a < 0$ and $\operatorname{Re} k > 0$.

6.532.5: $\dfrac{K_0(-ak)}{k}$ is the solution for $a > 0,\ \operatorname{Re} k < 0$.

6.532.6: $\dfrac{\pi}{2k}\left[I_0(-ak) - \boldsymbol{L}_0(-ak)\right]$ is the solution for $a < 0$, $\operatorname{Re} k > 0$ and $\dfrac{-\pi}{2k}\left[I_0(-ak)\right.$ $\left.-\boldsymbol{L}_0(-ak)\right]$ is the solution for $a > 0,\ \operatorname{Re} k < 0$. The solution for $a < 0,\ \operatorname{Re} k < 0$ is $\dfrac{-\pi}{2k}\left[I_0(ak)\ -\boldsymbol{L}_0(ak)\right]$.

6.533.3: the solution $-\dfrac{b}{4}\left[1 + 2\ln\left(\left|\dfrac{a}{b}\right|\right)\right]$ holds for $0 < b < a$, $a < b < 0$, $a < 0 < b$ with $a+b < 0$ and for $b < 0 < a$ with $a+b > 0$. The solution $-\dfrac{a^2}{4b}$ holds for $0 < a < b$, $b < a < 0$, $a < 0 < b$ with $a + b > 0$ and $b < 0 < a$ with $a + b < 0$.

6.554.1: the solution $y^{-1}e^{ay}$ holds for $y > 0$ and $\operatorname{Re} a < 0$. The solution $-y^{-1}e^{ay}$ holds for $y < 0$ and $\operatorname{Re} a > 0$. The solution $-y^{-1}e^{-ay}$ holds for $y < 0$ and $\operatorname{Re} a < 0$.

6.566.2: the solution also applies for $a < 0,\ \operatorname{Re} b < 0,\ -1 < \operatorname{Re}\nu < \frac{3}{2}$. For EH II 96(58), read EH II 96(58), ET II 23(12).

6.566.3: the solution $\dfrac{\pi^2(-b)^{\nu-1}}{4\cos\nu\pi}\left[\boldsymbol{H}_{-\nu}(-ab) - Y_{-\nu}(-ab)\right]$ holds for $a > 0$, $\operatorname{Re} b < 0$, $\operatorname{Re}\nu > -\frac{1}{2}$.

6.566.4: the solution $\dfrac{\pi^2}{4(-b)^{\nu+1}\cos\nu\pi}\left[\boldsymbol{H}_{\nu}(-ab) - Y_{\nu}(-ab)\right]$ holds for $a > 0$, $\operatorname{Re} b < 0$, $\operatorname{Re}\nu < \frac{1}{2}$.

6.566.5: the solution $\dfrac{-\pi}{2(-b)^{\nu+1}}\left[I_\nu(-ab) - \boldsymbol{L}_\nu(-ab)\right]$ is valid for $a < 0$, $\operatorname{Re} b > 0$, $\operatorname{Re}\nu > -\frac{5}{2}$ and the solution $\dfrac{\pi}{2(-b)^{\nu+1}}\left[I_\nu(-ab) - \boldsymbol{L}_\nu(-ab)\right]$ applies for $a > 0$, $\operatorname{Re} b < 0$, $\operatorname{Re}\nu > -\frac{5}{2}$. The solution $\dfrac{\pi}{2(-b)^{\nu+1}}\left[I_\nu(-ab) + \boldsymbol{L}_\nu(-ab)\right]$ holds for $a < 0$, $\operatorname{Re} b < 0$,$\operatorname{Re}\nu > -\frac{5}{2}$ as noted in Section 2.

6.592.7: for $\dfrac{1}{2}\sqrt{\pi}\sec(\nu\pi)$, read $\dfrac{1}{2}\pi\sec\left(\dfrac{1}{2}\nu\pi\right)$ as noted in [**14**]. For $|\operatorname{Re}\nu| < 1$, read $a \neq 0,\ |\operatorname{Re}\nu| < 1$.

6.611.2: for $\operatorname{Re}\alpha > 0,\ b > 0,\ |\operatorname{Re}\nu| < 1$, read $|\operatorname{Re}\nu| < 1,\ \operatorname{Re}(\alpha \pm ib) > 0$ as noted in Section 2.

6.613: for $\operatorname{Re}\nu > -1$, read $\operatorname{Re} z \geqslant 0,\ \operatorname{Re}\nu > -1$.

6.633.2: for $\operatorname{Re} p > -1,\ |\arg\rho| < \frac{\pi}{4},\ a > 0,\ \beta > 0$, read $\operatorname{Re} p > -1$, $|\arg\rho| < \frac{\pi}{4}$, a real, $\beta > 0$.

6.648: for $\left(\dfrac{a + be^x}{ae^x + b}\right)$, read $\left(\dfrac{a + be^x}{ae^x + b}\right)^{\nu}$.

6.671.7: add the solution ∞ for $a = b$ as noted on p. 405 in [**21**].

6.681.12: for $\frac{\pi}{2}$, read $\frac{\pi^2}{4}$. For $|\mathrm{Re}\,(u+\nu)| < 1$, read $a \neq 0,\ |\mathrm{Re}\,(u+\nu)| < 1$.

6.686.5: for $a > 0,\ b > 0$, read $a \neq 0,\ b \neq 0$ as noted in Section 2.

6.772.1: add $a > 0$. For negative a, the solution is $\frac{1}{a}\left[\ln(-2a) + \boldsymbol{C}\right],\ a > 0$.

6.772.2: add $a > 0$. For negative a, the solution is $-\frac{1}{a}\left[\ln\left(\frac{-a}{2}\right) + \boldsymbol{C}\right],\ a < 0$.

6.772.3: for $\frac{2}{b}\left[K_0(ab) + \ln a\right]$, read $\frac{2}{b}\left[K_0(|ab|) + \ln(|a|)\right]$, $a \neq 0,\ b \neq 0$.

6.772.4: add $x > 0$. For negative values of x, the solution is $\frac{2}{x}\,\mathrm{ker}\,(-x),\ x < 0$.

6.784.1: the solution is wrong. The correct solution is given in (8) on p. 15 in [**17**]

$$\frac{1}{2\sqrt{\pi}}\left(\frac{b}{2}\right)^{\nu}\frac{1}{a^{2\nu+2}}\frac{\Gamma\left[\nu+\frac{3}{2}\right]}{\Gamma\left[\nu+2\right]}\Phi\left(\nu+\frac{3}{2},\nu+2;-\frac{b^2}{4a^2}\right),$$

where $\Phi(\alpha,\beta;z)$ is the confluent hypergeometric function.
For $|\arg a| < \frac{\pi}{4},\ b > 0,\ \mathrm{Re}\,\nu > -1$, read $|\arg a| < \frac{\pi}{4},\ b \neq 0,\ \mathrm{Re}\,\nu > -1$.

6.794.9: for $2^{5/2}$, read $2^{7/2}$.

6.812.1: the solution for $\mathrm{Re}\,a > 0,\ b < 0$ is $\frac{\pi}{2a}\left[I_1(-ab) - \mathbf{L}_1(-ab)\right]$. For $\mathrm{Re}\,a < 0,\ b > 0$, the solution is $-\frac{\pi}{2a}\left[I_1(-ab) - \mathbf{L}_1(-ab)\right]$ and the solution for $\mathrm{Re}\,a < 0,\ b < 0$ is $-\frac{\pi}{2a}\left[I_1(ab) - \mathbf{L}_1(ab)\right]$.

6.812.2: for $\frac{a^2b^2}{2}$, read $\frac{a^2b^2}{4}$.

6.813.4: for $a > 0,\ \mathrm{Re}\,\nu > -\frac{3}{2}$, read $a \neq 0,\ \mathrm{Re}\,\nu > -\frac{3}{2}$.

6.813.5: for $a > 0$, read $a \neq 0$.

6.876.1: for x kei $x\ J_1(ax)$, read kei $x\ J_1(ax)$; for $a > 0$, read $a \neq 0$.

6.876.2: for x ker $x\ J_1(ax)$, read ker $x\ J_1(ax)$; for $a > 0$ read $a \neq 0$.

7.251.3: for $y > 0$, read $y \neq 0$.

7.355.1: $a > 0$ is to be deleted.

7.355.2: $a > 0$ is to be deleted.

7.374.4: for $e^{-x^2}H_{2m+n}(ax)\,H_n(x)$, read $e^{-\frac{1}{2}x^2}He_{2m+n}(ax)\,He_n(x)$.

7.374.7: for $L_n^{n-m}\left(-2y^2\right)$, read $L_m^{n-m}\left(-2y^2\right)$ as noted in [**16**]. The condition $m \leqslant n$ is to be deleted.

7.376.3: for $\Gamma\left(\frac{\nu+1}{2}\right)\Gamma\left(n+\frac{1}{2}\right)$, read $\Gamma\left(\frac{\nu}{2}+1\right)\Gamma\left(n+\frac{3}{2}\right)$.

7.421.1: for $y > 0,\ \mathrm{Re}\,\alpha > 0$, read $\mathrm{Re}\,\alpha > 0$.

7.512.6: for $B(\lambda,\beta-\lambda)\,F(\alpha,\lambda;\gamma;z)$, read $B(\lambda,\beta-\lambda)\,F(\alpha,\lambda;\lambda;z) = B(\lambda,\beta-\lambda)\,(1-z)^{-\alpha}$ (which corrects a misprint in the solution that was advanced in [**14**]).

7.531.1: for $\mu > 0,\ \mathrm{Re}\,\alpha > \frac{1}{2},\ \mathrm{Re}\,\beta > \frac{1}{2}$, read $\mu > 0,\ \mathrm{Re}\,\alpha > \frac{1}{2},\ \mathrm{Re}\,\beta > \frac{1}{2},\ c > 0$. This integral has been derived in [**3**].

7.662.4: the solution for negative a is $-ay^{-2}\left[\Gamma\left(2\mu+1\right)\right]^{2} W_{-\mu,k}\left(\frac{iy^{2}}{4a}\right), W_{-\mu,k}\left(-\frac{iy^{2}}{4a}\right)$ $a<0, \operatorname{Re} y>0,\ \operatorname{Re}\mu>-\frac{1}{2}$.

7.731.1: for $\operatorname{Re}^{2} a>0$, read $\operatorname{Re} a^{2}>0$.

7.751.1: for $y>0$, read $y\neq 0,\ a\neq 0, n=1,3,5,7,....$

7.751.2: for $y>0, |\arg a|<\frac{1}{4}\pi$, read $y\neq 0,\ a\neq 0$.

7.751.3: the integral can be rewritten as

$$\int_{0}^{\infty} J_{0}\left(xy\right) D_{\nu}\left(ax\right) D_{\nu+1}\left(ax\right) dx=\frac{1}{y}\left[\frac{\sqrt{\pi}}{\sqrt{2}\Gamma\left[-\nu\right]}-D_{\nu}\left(\frac{y}{a}\right) D_{\nu+1}\left(-\frac{y}{a}\right)\right],\ y\neq 0,\ a>0,$$

$$=-\frac{1}{y} D_{n}\left(\frac{y}{a}\right) D_{n+1}\left(-\frac{y}{a}\right),\ y\neq 0,\ a\neq\ 0, n=0,1,2,...,$$

as discussed in Section 3.

7.752.1: for $y>0,\ \operatorname{Re}\nu>-\frac{1}{2}$, read $y\neq 0,\ \operatorname{Re}\nu>-\frac{1}{2}$.

7.752.3: for $y>0,\ \operatorname{Re}\nu>-1$, read $y\neq 0,\ \operatorname{Re}\nu>-1$.

7.752.4: for $y>0,\ \operatorname{Re}\nu>-\frac{1}{2}$, read $y\neq 0,\ \operatorname{Re}\nu>-\frac{1}{2}$.

7.752.5: for $\operatorname{Re}\nu>-1,\ y>0$, read $y\neq 0,\ \operatorname{Re}\nu>-1$.

7.752.10: for $y>0,\ |\arg a|<\frac{1}{4}\pi,\ \operatorname{Re}\nu>-\frac{1}{2}$, read $y\neq 0,\ |\arg a|<\frac{1}{4}\pi,\ \operatorname{Re}\nu>-\frac{1}{2}$.

7.752.12: for $y>0, |\arg a|<\frac{1}{4}\pi,\ \operatorname{Re}\nu>-1$, read $y\neq 0,\ |\arg a|<\frac{1}{4}\pi,\ \operatorname{Re}\nu>-1$.

7.755.1: for $2^{-3/2}$, read $2^{-1/2}$; for $y>0,\ \operatorname{Re} a>0$, read $y\neq 0,\ \operatorname{Re} a>0$.

7.771: for $|a|<\frac{1}{2}\pi$, read $|a|<\frac{1}{2}\pi, \beta>0$; for $|a|>\frac{1}{2}\pi$, read $|a|>\frac{1}{2}\pi, \beta>0$; for ET II 298(22), read ET II 398(22).

8.250.5: add $\operatorname{Re} p>0,\ y>0$.

8.250.8: for $\Phi\left(-\frac{x^{2}}{2}\right)$ in the solution, read $\exp\left(-\frac{p^{2}}{2}\right)$.

8.250.9: for $\sqrt{\pi}\Phi\left(a\right)\Phi\left(b\right)$, read $-\sqrt{\pi}\Phi\left(a\right)\Phi\left(b\right)$.

8.253.1: for $F_{1}\left(1;\frac{3}{2};x^{2}\right)$, read ${}_{1}F_{1}\left(1;\frac{3}{2};x^{2}\right)$.

8.258.3: for $\left(1+\beta\right)\left(\beta^{2}+2\beta+2\right)$, read $\left(1+\beta\right)\left(2+\beta\right)$. A recurrence relation for **8.258.1-8.258.5** is derived in [**2**].

8.258.5: for $1\arctan\left(\sqrt{\beta}\right)$, read $\arctan\left(\sqrt{\beta}\right)$.

9.221: add $\operatorname{Re}\left(\mu\pm\lambda\right)>-\frac{1}{2}$. The integral then is also consistent with (16) on p. 274 of [**5**] in view of the identity $B\left(x,y\right)=\frac{\Gamma\left(x\right)\Gamma\left(y\right)}{\Gamma\left(x+y\right)}$ in **8.384**.

9.245.1: for x is real, $\operatorname{Re} p<0$, read $x\geqslant 0,\ \operatorname{Re} p<0$.

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