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Certain Triple Integral Relations Involving Multivariable H-function

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Abstract. The aim of this paper is to evaluate certain triple integral relations involving H-function and the multivariable H-function. Next, we give three Theorems containing the product of H-function, with the help of our main findings and using the Mellin integral transform. The results obtained here are quite general in nature due to the presence of functions which are basic in nature. A large number of new results have been obtained by proper choice of parameter.

1. Introduction

The H-function [5] occurring in this paper will be defined and represented in the following manner:

$$H_{P,Q}^{M,N}[z] = H_{P,Q}^{M,N}\left(z \,\middle|\, \begin{matrix}(a_j,\alpha_j)_{1,P}\\(b_j,\beta_j)_{1,Q}\end{matrix}\right) = \frac{1}{2\pi i}\int_L \varphi(\xi) z^{\xi} d\xi \tag{1.1}$$

where

$$\varphi(\xi) = \frac{\prod_{j=1}^{M}\Gamma(b_j-\beta_j\xi)\prod_{j=1}^{N}(1-a_j+\alpha_j\xi)}{\prod_{j=M+1}^{Q}\Gamma(1-b_j+\beta_j\xi)\prod_{j=N+1}^{P}\Gamma(a_j-\alpha_j\xi)} \tag{1.2}$$

For the convergence conditions and other details about this function we may refer to [5].
The multivariable H-function due to Srivastava and Panda [3] is defined and represented as follows:

$$H[z_1,...,z_r] = H_{A,C:(B',D');...;(B^{(r)},D^{(r)})}^{0,\delta:(U',V');...;(U^{(r)},V^{(r)})}\left[\begin{matrix}z_1\\ \vdots\\ z_r\end{matrix}\,\middle|\,\begin{matrix}\left[(a);\theta',...,\theta^{(r)}\right]:\left[\left(b'\right);\varphi'\right];...;\left[\left(b^{(r)}\right);\varphi^{(r)}\right]\\ \left[(c);\psi',...,\psi^{(r)}\right]:\left[\left(d'\right);\delta'\right];...;\left[\left(d^{(r)}\right);\delta^{(r)}\right]\end{matrix}\right]$$

$$= \frac{1}{(2\pi i)^r}\int_{L_1}...\int_{L_r}\psi(s_1,...,s_r)\varphi_1(s_1)...\varphi_r(s_r)z^{s_1}...z^{s_r}ds_1...ds_r \qquad \left(i=\sqrt{-1}\right) \tag{1.3}$$

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where

$$\varphi_i\left(s_i\right)=\frac{\prod\limits_{j=1}^{U^{(i)}}\Gamma(d_j^{(i)}-\delta_j^{(i)}s_i)\prod\limits_{j=1}^{V^{(i)}}\Gamma(1-b_j^{(i)}+\varphi_j^{(i)}s_i)}{\prod\limits_{j=m_i+1}^{q_i}\Gamma(1-d_j^{(i)}+\delta_j^{(i)}s_i)\prod\limits_{j=n_i+1}^{p_i}\Gamma(b_j^{(i)}-\varphi_j^{(i)}s_i)}\quad\forall i\in\{1,...,r\}\tag{1.4}$$

$$\psi\left(s_1,...,s_r\right)=\frac{\prod\limits_{j=1}^{\delta}\Gamma(1-a_j+\sum\limits_{i=1}^{r}\theta_j^{(i)}s_i)}{\prod\limits_{j=1}^{C}\Gamma(1-c_j+\sum\limits_{i=1}^{r}\psi_j^{(i)}s_i)\prod\limits_{j=\delta+1}^{A}\Gamma(a_j-\sum\limits_{i=1}^{r}\theta_j^{(i)}s_i)}\tag{1.5}$$

For the nature of contours, various sets of convergence conditions of the integral given by (1.3) and the other details about this function we may refer to [5].
For the sake of brevity, we used the following notations:

$$\zeta=\frac{\pi}{2^{2+\sigma}}H_{P,Q}^{M,N}\left[\alpha^{'}2^{-k}\left|\begin{array}{c}(a_j,\alpha_j)_{1,P}\\(b_j,\beta_j)_{1,Q}\end{array}\right.\right]\tag{1.6}$$

$$\eta=\frac{\pi}{\mu 2^{3+\sigma}}\beta^{-\left(\frac{2\lambda+1}{\mu}\right)}H_{P,Q}^{M,N}\left[\alpha^{'}2^{-k}\left|\begin{array}{c}(a_j,\alpha_j)_{1,P}\\(b_j,\beta_j)_{1,Q}\end{array}\right.\right]\tag{1.7}$$

2. The Main Tripple Integral Relations :

Our main results of the present paper are the triple integral relations contained in the following theorems.

Theorem 1.

$$\int_0^\infty\int_0^\infty\int_0^\infty\frac{(tu)^\sigma}{(t^2+u^2+z^2)^\sigma}\cos\left(2\lambda\tan^{-1}\frac{z}{t}\right)H_{P,Q}^{M,N}\left[\frac{\alpha^{'}(tu)^k}{(t^2+u^2+z^2)^k}\left|\begin{array}{c}(a_j,\alpha_j)_{1,P}\\(b_j,\beta_j)_{1,Q}\end{array}\right.\right]$$

$$H_{A,C:\left(B^{'},D^{'}\right);...;\left(B^{(r)},D^{(r)}\right)}^{0,\delta:\left(U^{'},V^{'}\right);...;\left(U^{(r)},V^{(r)}\right)}\left[\begin{array}{c}y_1\left(tu\right)^{\tau}\left(t^2+u^2+z^2\right)^{b_1-\varsigma}\\y_2\left(t^2+u^2+z^2\right)^{b_2}\\\vdots\\y_r\left(t^2+u^2+z^2\right)^{b_r}\end{array}\right]f\left(t^2+u^2+z^2\right)dt\,du\,dz$$

$$=\zeta\int_0^\infty H_{A,C:(B'+3,D'+3)*}^{0,\delta:\left(U',V'+3\right)*}\left[\begin{array}{c}y_1\rho^{2b_1}2^{-\tau}\\y_2\rho^{2b_2}\\\vdots\\y_r\rho^{2b_1}\end{array}\left|\begin{array}{c}\left[(a);\theta^{'},...,\theta^{(r)}\right]:\left[\frac{1}{2}-\frac{\sigma}{2}-\frac{k\xi}{2};\frac{\tau}{2}\right]\left[-\frac{\sigma}{2}-\frac{k\xi}{2};\tau\right]\left[-\sigma-\frac{k\xi}{2};\tau\right]*\\\left[(c);\psi^{'},...,\psi^{(r)}\right]:\left[-\frac{1}{2}-\sigma-k\xi;\tau\right]\left[-\frac{\sigma}{2}-\frac{k\xi}{2}\pm\frac{\lambda}{2};\tau\right]*\end{array}\right.\right]$$

$$\times\rho^2 f\left(\rho^2\right)d\rho\tag{2.1}$$

where

$$\left[\mathrm{Re}\left(\sigma\right)+ks\min\mathrm{Re}\left(\frac{b_j}{\beta_j}\right)-\sum_{i=1}^{r}b_i\xi_i\max\mathrm{Re}\left(\frac{b_j^{(i)}-1}{\varphi_j^{(i)}}\right)+\tau\xi_1\min\mathrm{Re}\left(\frac{d_j^{'}}{\delta_j^{'}}\right)-\frac{1}{2}\right]>0$$

$$\left[\mathrm{Re}\left(\sigma\right)+ks\min\mathrm{Re}\left(\frac{b_j}{\beta_j}\right)+\tau\xi_1\mathrm{Re}\min\left(\frac{d_j^{'}}{\delta_j^{'}}\right)+1\right]>0$$

$$\left[\mathrm{Re}\,(\sigma)+ks\min\mathrm{Re}\left(\frac{b_j}{\beta_j}\right)-2\sum_{i=1}^{r}b_i\xi_i\max\mathrm{Re}\left(\frac{b_j^{(i)}-1}{\varphi_j^{(i)}}\right)+\tau\xi_1\min\mathrm{Re}\left(\frac{d_j'}{\delta_j'}\right)-1\right]>0$$

where the asterisk * in (2.1) indicates that the parameters at these places are the same as the parameters of the H- function of several variables defined by (1.3).

Theorem 2.

$$\int_0^\infty\int_0^\infty\int_0^\infty \frac{(tu)^\sigma}{(t^2+u^2+z^2)^\sigma}\cos\left(2\lambda\tan^{-1}\frac{z}{t}\right)H_{P,Q}^{M,N}\left[\frac{\alpha'(tu)^\sigma}{(t^2+u^2+z^2)^\sigma}\left|\begin{matrix}(a_j,\alpha_j)_{1,P}\\(b_j,\beta_j)_{1,Q}\end{matrix}\right.\right]$$

$$H_{A,C:(B',D');\ldots;(B^{(r)},D^{(r)})}^{0,\delta:(U',V');\ldots;(U^{(r)},V^{(r)})}\left[\begin{matrix}y_1(tu)^{-\tau}\left(t^2+u^2+z^2\right)^{b_1+\varsigma}\\ y_2\left(t^2+u^2+z^2\right)^{b_2}\\ \vdots\\ y_r\left(t^2+u^2+z^2\right)^{b_r}\end{matrix}\right]f\left(t^2+u^2+z^2\right)dt\,du\,dz$$

$$=\zeta\int_0^\infty H_{A,C:(B'+3,D'+3);*}^{0,\delta:(U'+3,V');*}\left[\begin{matrix}y_1\rho^{2b_1}2^{-\tau}\\ y_2\rho^{2b_2}\\ \vdots\\ y_r\rho^{2b_1}\end{matrix}\left|\begin{matrix}\left[(a);\theta',\ldots,\theta^{(r)}\right]:\left[1+\frac{\sigma}{2}+\frac{k\xi}{2};\frac{\tau}{2}\right]\left[\frac{1}{2}+\frac{\sigma}{2}+\frac{k\xi}{2};\frac{\tau}{2}\right][1+\sigma+k\xi;\tau]*\\ \left[(c);\psi',\ldots,\psi^{(r)}\right]:\left[\frac{3}{2}+\sigma+k\xi;\tau\right]\left[1+\frac{\sigma}{2}+\frac{k\xi}{2}\pm\lambda;\tau\right]*\end{matrix}\right.\right]$$

$$\times\rho^2 f\left(\rho^2\right)d\rho \qquad (2.2)$$

valid under the conditions as obtainable from (2.1).

Theorem 3.

$$\int_0^\infty\int_0^\infty\int_0^\infty \frac{(tu)^\sigma}{(t^2+u^2+z^2)^\sigma}\cos\left(2\lambda\tan^{-1}\frac{z}{t}\right)H_{P,Q}^{M,N}\left[\frac{\alpha'(tu)^\sigma}{(t^2+u^2+z^2)^\sigma}\left|\begin{matrix}(a_j,\alpha_j)_{1,P}\\(b_j,\beta_j)_{1,Q}\end{matrix}\right.\right]$$

$$H_{A,C:(B',D');\ldots;(B^{(r)},D^{(r)})}^{0,\delta:(U',V');\ldots;(U^{(r)},V^{(r)})}\left[\begin{matrix}y_1(tu)^{\tau_1}\left(t^2+u^2+z^2\right)^{b_1-\tau_1}\\ y_2(tu)^{\tau_2}\left(t^2+u^2+z^2\right)^{b_2-\tau_2}\\ \vdots\\ y_r(tu)^{\tau_r}\left(t^2+u^2+z^2\right)^{b_r-\tau_r}\end{matrix}\right]f\left(t^2+u^2+z^2\right)dt\,du\,dz$$

$$=\zeta\int_0^\infty H_{A+3,C+3:*}^{0,\delta+3:*}\left[\begin{matrix}y_1\rho^{2b_1}2^{-\tau_1}\\ y_2\rho^{2b_2}2^{-\tau_2}\\ \vdots\\ y_r\rho^{2b_1}2^{-\tau_r}\end{matrix}\left|\begin{matrix}\left[(a);\theta',\ldots,\theta^{(r)}\right]\left[1+\frac{\sigma}{2}+\frac{k\xi}{2};\frac{\tau}{2}\right]\left[\frac{1}{2}+\frac{\sigma}{2}+\frac{k\xi}{2};\frac{\tau}{2}\right][1+\sigma+k\xi;\tau]:*\\ \left[(c);\psi',\ldots,\psi^{(r)}\right]\left[\frac{3}{2}+\sigma+k\xi;\tau\right]\left[1+\frac{\sigma}{2}+\frac{k\xi}{2}\pm\lambda;\tau\right]:*\end{matrix}\right.\right]$$

$$\times\rho^2 f\left(\rho^2\right)d\rho \qquad (2.3)$$

where the conditions of validity are same as surrounding Theorem 1.

Proof: To prove Theorem 1, first we change the L.H.S. of integral (2.1) from cartesian to polar form and then expressing the H- function and multivariable H- function in their contour form with the help of (1.1) and (1.3). Now changing the order of integration, we get the following form of integral (say Δ)

$$\Delta = \frac{1}{(2\pi i)^r}\int_{L_1}\cdots\int_{L_2}\psi(\varphi_1,...,\varphi_r)\varphi_1(\xi_1)...\varphi_r(\xi_r)\,y_1^{\xi_1}...y_r^{\xi_r}\frac{1}{2\pi i}\int_L\varphi(\xi)\alpha'^{\xi}\left[\int_0^{\infty}\int_0^{\pi/2}\rho^{2+2b_1\xi_1+...+2b_r\xi_r}f\left(\rho^2\right)\right.$$

$$\left.(\sin\theta)^{\sigma+k\xi+\tau\xi_1+1}(\cos\theta)^{\sigma+k\xi+\tau\xi_1}\left\{\int_0^{\pi/2}(\cos\varphi)^{\sigma+k\xi+\tau\xi_1}\cos(2\lambda\varphi)\,d\varphi\right\}d\theta d\rho\right]d\xi.d\xi_1...d\xi_r \tag{2.4}$$

On evaluating the θ and ϕ integral occurring on the R. H. S. of (2.4) with the help of known result [1, Eq. 5, p.16], see also[2, Eq.3.2.7, p. 62] and using the well known Beta function, we easily arrive at the desired result (2.1) after a little simplification.

Theorems 2 and 3 can be proved on the similar lines.

Application:

In the triple integral (2.1) through (2.3), if we take

$$f\left(\rho^2\right) = \rho^{2\lambda-2}H_{p,q}^{m,0}\left[\beta\rho^{2\mu}\left|\begin{matrix}(e_p,E_p)\\(f_q,F_q)\end{matrix}\right.\right] \tag{2.5}$$

and evaluate the ρ integral by means of the known result [5, Eq.2.4.1 , p.15], we have the following triple integral relations.

Theorem 4.

$$\int_0^{\infty}\int_0^{\infty}\int_0^{\infty}\frac{(tu)^{\sigma}}{(t^2+u^2+z^2)^{\sigma-\lambda+1}}\cos\left(2\lambda\tan^{-1}\frac{z}{t}\right)H_{P,Q}^{M,N}\left[\frac{\alpha'(tu)^k}{(t^2+u^2+z^2)^k}\left|\begin{matrix}(a_j,\alpha_j)_{1,P}\\(b_j,\beta_j)_{1,Q}\end{matrix}\right.\right]$$

$$H_{p,q}^{m,0}\left[\beta\left(t^2+u^2+z^2\right)^{\mu}\left|\begin{matrix}(e_p,E_p)\\(f_q,F_q)\end{matrix}\right.\right]H_{A,C:(B',D');...;(B^{(r)},D^{(r)})}^{0,\delta:(U',V');...;(U^{(r)},V^{(r)})}\left[\begin{matrix}y_1(tu)^{\tau}\left(t^2+u^2+z^2\right)^{b_1-\varsigma}\\y_2\left(t^2+u^2+z^2\right)^{b_2}\\\vdots\\y_r\left(t^2+u^2+z^2\right)^{b_r}\end{matrix}\right]dt\,du\,dz$$

$$=\eta H_{A+q,C+p:(B'+3,D'+3);*}^{0,\delta:(U',V'+3);*}\left[\begin{matrix}y_1\beta^{-b_1/\mu}2^{-\tau}\\y_2\beta^{-b_2/\mu}\\\vdots\\y_r\beta^{-b_r/\mu}\end{matrix}\left|\begin{matrix}\left[(a);\theta',...,\theta^{(r)}\right]\left[1-f_j-F_j\frac{(2\lambda+1)}{2\mu};F_j\frac{b_1}{\mu},...,F_j\frac{b_r}{\mu}\right]_{1,q}\\\left[(c);\psi',...,\psi^{(r)}\right]\left[1-e_j-E_j\frac{(2\lambda+1)}{2\mu};E_j\frac{b_1}{\mu},...,E_j\frac{b_r}{\mu}\right]_{1,p}\end{matrix}\right.\right.$$

$$\left.\begin{matrix}\left[\frac{1}{2}-\frac{\sigma}{2}-\frac{k\xi}{2};\frac{\tau}{2}\right]\left[-\frac{\sigma}{2}-\frac{k\xi}{2};\tau\right]\left[-\sigma-\frac{k\xi}{2};\tau\right]*\\\left[-\frac{1}{2}-\sigma-k\xi;\tau\right]\left[-\frac{\sigma}{2}-\frac{k\xi}{2}\pm\frac{\lambda}{2};\tau\right]*\end{matrix}\right] \tag{2.6}$$

where

$$\Big[\mathrm{Re}\,(\sigma)-\lambda+ks\min\mathrm{Re}\left(\tfrac{b_j}{\beta_j}\right)-\mu\max\mathrm{Re}\left(\tfrac{e_j-1}{E_j}\right)$$
$$2\sum_{i=1}^{r}b_i\xi_i\max\left(\tfrac{b_j^{(i)}-1}{\varphi_j^{(i)}}\right)+\tau\xi_1\min\mathrm{Re}\left(\tfrac{d_j'}{\delta_j'}\right)+\tfrac{1}{2}\Big]>0$$
$$\Big[\mathrm{Re}\,(\sigma)+ks\min\mathrm{Re}\left(\tfrac{b_j}{\beta_j}\right)+\tau\min\mathrm{Re}\left(\tfrac{d_j'}{\delta_j'}\right)+1\Big]>0$$
$$\Big[\mathrm{Re}\,(\sigma-2\lambda)+ks\min\mathrm{Re}\left(\tfrac{b_j}{\beta_j}\right)-2\mu\max\mathrm{Re}\left(\tfrac{e_j-1}{E_j}\right)$$
$$-2\sum_{i=1}^{r}b_i\xi_i\max\left(\tfrac{b_j^{(i)}-1}{\varphi_j^{(i)}}\right)+\tau\xi_1\min\mathrm{Re}\left(\tfrac{d_j'}{\delta_j'}\right)+1\Big]>0$$

Theorem 5.

$$\int_0^\infty\int_0^\infty\int_0^\infty\frac{(tu)^\sigma}{(t^2+u^2+z^2)^{\sigma-\lambda+1}}\cos\left(2\lambda\tan^{-1}\frac{z}{t}\right)H_{P,Q}^{M,N}\left[\frac{\alpha'(tu)^k}{(t^2+u^2+z^2)^k}\middle|\begin{matrix}(a_j,\alpha_j)_{1,P}\\(b_j,\beta_j)_{1,Q}\end{matrix}\right]$$

$$H_{p,q}^{m,0}\left[\beta\left(t^2+u^2+z^2\right)^\mu\middle|\begin{matrix}(e_p,E_p)\\(f_q,F_q)\end{matrix}\right]H_{A,C:(B',D');\dots;(B^{(r)},D^{(r)})}^{0,\delta:(U',V');\dots;(U^{(r)},V^{(r)})}\begin{bmatrix}y_1(tu)^{-\tau}\left(t^2+u^2+z^2\right)^{b_1+\varsigma}\\y_2\left(t^2+u^2+z^2\right)^{b_2}\\\vdots\\y_r\left(t^2+u^2+z^2\right)^{b_r}\end{bmatrix}dt\,du\,dz$$

$$=\eta H_{A+q,C+p:(B'+3,D'+3)*}^{0,\delta:(U'+3,V')*}\left[\begin{matrix}y_1\beta^{-b_1/\mu}2^{-\tau}\\y_2\beta^{-b_2/\mu}\\\vdots\\y_r\beta^{-b_r/\mu}\end{matrix}\middle|\begin{matrix}\left[(a);\theta',\dots,\theta^{(r)}\right]\left[1-f_j-F_j\frac{(2\lambda+1)}{2\mu};F_j\frac{b_1}{\mu},\dots,F_j\frac{b_r}{\mu}\right]_{1,q}\\\left[(c);\psi',\dots,\psi^{(r)}\right]\left[1-e_j-E_j\frac{(2\lambda+1)}{2\mu};E_j\frac{b_1}{\mu},\dots,E_j\frac{b_r}{\mu}\right]_{1,p}\end{matrix}\right.$$

$$\left.\begin{matrix}\left[1+\frac{\sigma}{2}+\frac{k\xi}{2};\frac{\tau}{2}\right]\left[\frac{1}{2}+\frac{\sigma}{2}+\frac{k\xi}{2};\frac{\tau}{2}\right][1+\sigma+k\xi;\tau]*\\\left[\frac{3}{2}+\sigma+k\xi;\tau\right]\left[1+\frac{\sigma}{2}+\frac{k\xi}{2}\pm\lambda;\tau\right]*\end{matrix}\right]\tag{2.7}$$

Theorem 6.

$$\int_0^\infty\int_0^\infty\int_0^\infty\frac{(tu)^\sigma}{(t^2+u^2+z^2)^{\sigma-\lambda+1}}\cos\left(2\lambda\tan^{-1}\frac{z}{t}\right)H_{P,Q}^{M,N}\left[\frac{\alpha'(tu)^k}{(t^2+u^2+z^2)^k}\middle|\begin{matrix}(a_j,\alpha_j)_{1,P}\\(b_j,\beta_j)_{1,Q}\end{matrix}\right]$$

$$H_{p,q}^{m,0}\left[\beta\left(t^2+u^2+z^2\right)^\mu\middle|\begin{matrix}(e_p,E_p)\\(f_q,F_q)\end{matrix}\right]H_{A,C:(B',D');\dots;(B^{(r)},D^{(r)})}^{0,\delta:(U',V');\dots;(U^{(r)},V^{(r)})}\begin{bmatrix}y_1(tu)^{\tau_1}\left(t^2+u^2+z^2\right)^{b_1-\tau_1}\\\vdots\\y_r(tu)^{\tau_r}\left(t^2+u^2+z^2\right)^{b_r-\tau_r}\end{bmatrix}dt\,du\,dz$$

$$=\eta H_{A+q+3,C+p+3:*}^{0,\delta+3:*}\left[\begin{matrix}y_1\beta^{-b_1/\mu}2^{-\tau_1}\\\vdots\\y_r\beta^{-b_r/\mu}2^{-\tau_r}\end{matrix}\middle|\begin{matrix}\left[(a);\theta',\dots,\theta^{(r)}\right]\left[1-f_j-F_j\frac{(2\lambda+1)}{2\mu};F_j\frac{b_1}{\mu},\dots,F_j\frac{b_r}{\mu}\right]_{1,q}\\\left[(c);\psi',\dots,\psi^{(r)}\right]\left[1-e_j-E_j\frac{(2\lambda+1)}{2\mu};E_j\frac{b_1}{\mu},\dots,E_j\frac{b_r}{\mu}\right]_{1,p}\end{matrix}\right.$$

$$\left.\begin{matrix} \left[-\frac{\sigma}{2}-\frac{k\xi}{2};\frac{\tau_1}{2},...,\frac{\tau_r}{2}\right]\left[\frac{1}{2}-\frac{\sigma}{2}-\frac{k\xi}{2};\frac{\tau_1}{2},...,\frac{\tau_r}{2}\right][-\sigma-k\xi;\tau_1,...,\tau_r]* \\ \left[-\frac{1}{2}-\sigma-k\xi;\tau_1,...,\tau_r\right]\left[-\frac{\sigma}{2}-\frac{k\xi}{2}\pm\lambda;\tau_1,...,\tau_r\right]* \end{matrix}\right] \tag{2.8}$$

where the conditions of validity are same as of Theorem 4.

Particular Cases: If we reduce the multivariable H-function into generalized Lauricella function defined by Srivastava and Daoust [4] in our result (2.1), we get the following interesting consequence of our main result:

$$\int_0^\infty\int_0^\infty\int_0^\infty \frac{(tu)^\sigma}{(t^2+u^2+z^2)^\sigma}\cos\left(2\lambda\tan^{-1}\frac{z}{t}\right)H_{P,Q}^{M,N}\left[\frac{\alpha'(tu)^k}{(t^2+u^2+z^2)^k}\left|\begin{matrix}(a_j,\alpha_j)_{1,P}\\(b_j,\beta_j)_{1,Q}\end{matrix}\right.\right]$$
$$F_{C:D';...;D^{(r)}}^{A:B';...;B^{(r)}}\left[\begin{matrix} y_1(tu)^\tau\left(t^2+u^2+z^2\right)^{b_1-\varsigma}\\ y_2\left(t^2+u^2+z^2\right)^{b_2}\\ \vdots\\ y_r\left(t^2+u^2+z^2\right)^{b_r}\end{matrix}\left|\begin{matrix}\left(1-a_j:\theta';...;\theta^{(r)}\right)\\ \left(1-c_j:\psi';...;\psi^{(r)}\right)\end{matrix}\right.\right.$$
$$\left.\begin{matrix}\left(1-b_j':\varphi'\right)_{1,B'};...;\left(1-b_j^{(r)}:\varphi^{(r)}\right)_{1,B^{(r)}}\\ \left(1-d_j':\delta'\right)_{1,B'};...;\left(1-d_j^{(r)}:\delta^{(r)}\right)_{1,B^{(r)}}\end{matrix}\right]f\left(t^2+u^2+z^2\right)dt\,du\,dz$$
$$=\Omega\int_0^\infty F_{C:(D'+3);D'';...;D^{(r)}}^{A:(B'+3);B'';...;B^{(r)}}\left[\begin{matrix}y_1\rho^{2b_1}2^{-\tau}\\ y_2\rho^{2b_2}\\ \vdots\\ y_r\rho^{2b_1}\end{matrix}\left|\begin{matrix}\left[1-a_j:\theta';...;\theta^{(r)}\right]\left[\frac{1}{2}+\frac{\sigma}{2}+\frac{k\xi}{2},\frac{\tau}{2}\right]\left[1+\frac{\sigma}{2}+\frac{k\xi}{2},\tau\right]\\ \left[1-c_j:\psi';...;\psi^{(r)}\right]\left[\frac{3}{2}+\sigma+k\xi,\tau\right]\end{matrix}\right.\right.$$
$$\left.\begin{matrix}\left[1+\sigma+\frac{k\xi}{2},\tau\right]*\left(1-b_j':\varphi'\right)_{1,B'};...;\left(1-b_j^{(r)}:\varphi^{(r)}\right)_{1,B^{(r)}}\\ \left[1+\frac{\sigma}{2}+\frac{k\xi}{2}\pm\frac{\lambda}{2},\tau\right]*\left(1-d_j':\delta'\right)_{1,B'};...;\left(1-d_j^{(r)}:\delta^{(r)}\right)_{1,B^{(r)}}\end{matrix}\right]\rho^2 f\left(\rho^2\right)d\rho \tag{2.9}$$

where

$$\Omega=\frac{\pi\Gamma\left(\frac{1}{2}+\frac{\sigma}{2}+\frac{k\xi}{2}\right)\Gamma\left(1+\frac{\sigma}{2}+\frac{k\xi}{2}\right)\Gamma\left(1+\sigma+\frac{k\xi}{2}\right)}{2^{2+\sigma}\Gamma\left(\frac{3}{2}+\sigma+\frac{k\xi}{2}\right)\Gamma\left(1+\frac{\sigma}{2}+\frac{k\xi}{2}\pm\frac{\lambda}{2}\right)}H_{P,Q}^{M,N}\left[\alpha'2^{-k}\left|\begin{matrix}(a_j,\alpha_j)_{1,P}\\(b_j,\beta_j)_{1,Q}\end{matrix}\right.\right] \tag{2.10}$$

A number of other special cases of our results can be obtained by reducing the H multivariable function by specializing its parameters, but we do not record them here explicitly.

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