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The integration of certain product involving special functions

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Abstract. The object of the present paper is to establish an integral pertaining to a product of Fox H-function [4], M-series [7], general polynomials [11] with general arguments of quadratic nature. This integral is unified in nature and capable of yielding a very large number of corresponding results (new and known) involving simpler special functions and polynomials as special cases of our integral.

1. Introduction

The M-series is defined by Sharma [7] as

$$
{}_p\overset{\gamma}{M}_q(u_1, ..., u_p; v_1, ..., v_q; w) =_p \overset{\gamma}{M}_q(w)
$$

$$
{}_p\overset{\gamma}{M}_q(w) = \sum_{k=0}^{\infty} \frac{(u_1)_k ...(u_p)_k}{(v_1)_k ...(v_q)_k} \frac{w^k}{\Gamma(\gamma k + 1)}, \tag{1.1}
$$

here $\gamma \in C, \mathrm{Re}(\gamma) > 0,$ and $(u_j)_k, (v_j)_k$ are the Poachemmar symbols. The series (1.1) is defined when none of the parameters v_js, j = l, 2,...,q is negative integer or zero. If any numerator parameter u_j is a negative or zero, the series terminates to a polynomial in w. From the ratio test it is evident that the series (1.1) is convergent for all w if $p \leqslant q$, if it convergent for $p = q + 1$ and divergent, if $p > q + 1$. When $p = q + 1$ and $|w| = 1$, the series can converge in some cases. Let $\beta = \sum\limits_{j=1}^{s} u_j - \sum\limits_{j=1}^{t} v_j$. It can be shown that when $p = q + 1$ the series is absolutely convergent for $|w| = 1$ if $Re(\beta) < 0$, conditionally convergent of $w = 1$ if $0 \leqslant Re(\beta) < 1$ and divergent for $|w| = 1$ if $1 \leqslant Re(\beta)$. The series representation of Fox H-function [9] :

$$
H_{P,Q}^{M,N}\left[z\left|{}^{(e_p,E_p)}_{(f_Q,F_Q)}\right.\right] = \sum_{G=0}^{\infty}\sum_{g=1}^{M} \frac{(-1)^G}{G! \quad F_g} \varphi(\eta_G) z^{\eta_G}, \tag{1.2}
$$

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where

$$\varphi(\eta_G) = \frac{\prod_{j=1, j\neq g}^{M} \Gamma(f_j - F_j\eta_G) \prod_{j=1}^{N} \Gamma(1 - e_j - E_j\eta_G)}{\prod_{j=M+1}^{Q} \Gamma(1 - f_j + F_j\eta_G) \prod_{j=N+1}^{P} \Gamma(e_j - E_j\eta_G)}$$

and

$$\eta_G = \frac{f_g + G}{F_g}$$

The H-function of several complex variables is defined by Srivastava and Panda [11] as :

$$H[z_1, ..., z_r] = H^{o,\lambda:(u',v');...;(u^{(r)},v^{(r)})}_{A,C:[B'D'];...;[B^{(r)},D^{(r)}]} \left[\begin{matrix} [(a):\theta';...;\theta^{(r)}]: \\ [(c):\psi';...;\psi^{(r)}]: \end{matrix} \right.$$

$$\left. \begin{matrix} [(b'):\varphi'];...;[(b^{(r)}):\varphi^{(r)}] \\ [(d'):\delta'];...;[(d^{(r)}):\delta^{(r)}] \end{matrix} \right| z_1, ..., z_r \Bigg] \tag{1.3}$$

The H-function of several complex variables in (1.3) converges absolutely if

$$|\arg(z_i)| < \frac{1}{2}\pi T_i, \tag{1.4}$$

$$T_i = \sum_{j=1}^{\lambda} \theta_j^{(i)} - \sum_{j=1}^{A} \theta_j^{(i)} + \sum_{j=1}^{v^{(i)}} \varphi_j^{(i)} - \sum_{j=v^{(i)}+1}^{B^{(i)}} \varphi_j^{(i)} - \sum_{j=1}^{C} \psi_j^{(i)} - \sum_{j=1}^{u^{(i)}} \delta_j^{(i)} - \sum_{j=u^{(i)}+1}^{D^{(i)}} \delta_j^{(i)} \tag{1.5}$$

$$\forall j \in (1, 2, ..., r)$$

Srivastava has defined and introduced the general polynomials [10] as

$$S^{M_1,...,M_S}_{N_1...,N_S}[w_1, ..., w_s] = \sum_{\alpha_1=0}^{[N_1/M_1]} ... \sum_{\alpha_s=0}^{[N_s/M_s]} \frac{(-N_1)_{M_1\alpha_1}}{\alpha_1!} ... \frac{(-N_s)_{M_s\alpha_s}}{\alpha_s!}$$

$$A[N_1, \alpha_1; ...; N_s, \alpha_s] w_1^{\alpha_1} ... w_s^{\alpha_s}, \tag{1.6}$$

where $N_i = 0, 1, 2, ..., i = (1, .., s)$;$M_1, ..., M_s$ are arbitrary positive integers and the coefficient $A[N_1, \alpha_1; ...; N_s, \alpha_s]$ are arbitrary constants, real or complex.

2. The Main Integral

The following integral has been derived here

$$\int_0^\infty t^{1-\alpha}(a + bt + ct^2)^{\alpha-3/2} H \begin{matrix} M, N \\ P, Q \end{matrix} \left[\left(\frac{t}{a + bt + ct^2} \right)^{\sigma} \Big|^{(e_p,E_p)}_{(f_Q,F_Q)} \right]$$

$$\cdot_p \overset{\gamma}{M}_q \left[w \left(\frac{t}{a + bt + ct^2} \right)^{n} \right] S \begin{matrix} M_1, ..., M_s \\ N_1, ..., N_s \end{matrix} \left[w_1 \left(\frac{t}{a + bt + ct^2} \right)^{n_1}, ..., w_s \left(\frac{t}{a + bt + ct^2} \right)^{n_s} \right]$$

$$H \left[z_1 \left(\frac{t}{a + bt + ct^2} \right)^{\sigma_1}, ..., z_r \left(\frac{t}{a + bt + ct^2} \right)^{\sigma_r} \right] dt$$

$$= \sqrt{\frac{\pi}{c}} \sum_{G,k=0}^{\infty} \sum_{g=1}^{M} \sum_{\alpha_1=0}^{[N_1/M_1]} ... \sum_{\alpha_s=0}^{[N_s/M_s]} \frac{(-1)^G (u_1)_k ... (u_p)_k}{G! F_g \quad (v_1)_k ... (v_q)_k} \frac{1}{\Gamma(\gamma k + 1)} \varphi(\eta_G) \frac{(-N_1)_{M_1\alpha_1}}{\alpha_1!} ... \frac{(-N_s)_{M_s\alpha_s}}{\alpha_s!}$$

$$A[N_1,\alpha_1;...;N_s,\alpha_s]w^k\ w_1^{\alpha_1}...w_s^{\alpha_s}(b+2\sqrt{ca})^{\alpha-\sigma\eta_G-nk-\sum_{i=1}^{s}n_i\alpha_i-1}$$

$$H\begin{matrix}0,\lambda+1 & :(u',v');...;(u^{(r)},v^{(r)})\\ A+1,C+1 & :[B^{(r)},D^{(r)}];...;[B^{(r)},D^{(r)}]\end{matrix}\left[\begin{matrix}z_1(b+2\sqrt{ca})^{-\sigma_1}\\ \vdots\\ z_r(b+2\sqrt{ca})^{-\sigma_r}\end{matrix}\left|\begin{matrix}[\alpha-\sigma\eta_G-nk-\sum_{i=1}^{s}n_i\alpha_i:\sigma_1;...;\sigma_r],\\ [(c);\psi',...,\psi^{(r)}],\end{matrix}\right.\right.$$

$$\left.\begin{matrix}[(a):\theta',....,\theta^{(r)}];[(b'):\varphi'];.....;[b^{(r)}):\varphi^{(r)}]\\ [\alpha-\sigma\eta_G-nk-\sum_{i=1}^{s}n_i\alpha_i-\frac{1}{2}:\sigma_1,...,\sigma_r]:[(d'):\delta'];.....;[d^{(r)}:\delta^{(r)}]\end{matrix}\right], \tag{2.1}$$

Provided that $Re(a)>0, Re(b)>0, c>0, p\leqslant q, |w|=1$ and

$$\sigma\ \min\left[\mathrm{Re}\left(\frac{f_j}{F_j}\right)\right]+\sum_{i'=1}^{r}\sigma_{i'}\ \min\left[\mathrm{Re}\left(\frac{d_{j'}^{(i')}}{\delta_{j'}^{(i')}}\right)\right]>\alpha-2, j=1,...,M\ and\ j'=1,...,u^{(i')}$$

Proof. In order to prove (2.1), we first express the Fox H-function, M-series and a general polynomials in the form of series and the H-function of several complex variables in terms of Mellin-Barnes contour integrals. Now interchanging the order of summations and integrations which is permissible under the stated conditions, we obtain

$$\sum_{G,k=0}^{\infty}\sum_{g=1}^{M}\sum_{\alpha_1=0}^{[N_1/M_1]}...\sum_{\alpha_s=0}^{[N_s/M_s]}\frac{(-1)^G(u_1)_k...(u_p)_k}{G!F_g(v_1)_k...(v_q)_k\Gamma(\gamma k+1)}\frac{(-N_1)_{M_1\alpha_1}}{\alpha_1!}...$$
$$.\frac{(-N_s)_{M_s\alpha_s}}{\alpha_s!}\varphi(\eta_G)A[N_1,\alpha_1;...;N_s,\alpha_s]w^kw_1^{\alpha_1}...w_s^{\alpha_s}\frac{1}{(2\pi i)^r}\int_{L_1}...\int_{L_r}\psi(\xi_1,...,\xi_r)$$
$$.\Phi_1(\xi_1)...\Phi_r(\xi_r)z_1^{\xi_1}.....z_r^{\xi_r}\left[\int_0^\infty t^{1-(\alpha-\sigma\eta_G-nk-\sum_{i=1}^{s}n_i\alpha_i-\sigma_1\xi_1-...-\sigma_r\xi_r)}\right. \tag{2.2}$$
$$\left..(a+bt+ct^2)^{(\alpha-\sigma\eta_G-nk-\sum_{i=1}^{s}n_i\alpha_i-\sigma_1\xi_1-...-\sigma_r\xi_r)-3/2}dt\right]d\xi_1....d\xi_r.$$

Evaluating the above t-integral with the help of a known theorem (Saxena [8]) and reinterpreting the result thus obtained in terms of H-function of r-variables, we arrive at the desired result.

Particular Cases:

I. Taking $\lambda=A, u^{(i)}=1, v^{(i)}=B^{(i)}\ and\ D^{(i)}=D^{(i)}+1, \forall i\in(1,...,r)$ the result in (2.1) reduced to the following integral transformation :

$$\int_0^\infty t^{1-\alpha}(a+bt+ct^2)^{\alpha-3/2}H_{P,Q}^{M,N}\left[\left(\frac{t}{a+bt+ct^2}\right)^\sigma\left|\begin{matrix}(e_P,E_P)\\(f_Q,F_Q)\end{matrix}\right.\right]$$
$$._p\overset{\gamma}{M}_q\left[w\left(\frac{t}{a+bt+ct^2}\right)^n\right]S\begin{matrix}M_1,...,M_s\\N_1,...,N_s\end{matrix}\left[w_1\left(\frac{t}{a+bt+ct^2}\right)^{n_1},...,w_s\left(\frac{t}{a+bt+ct^2}\right)^{n_s}\right]$$
$$.F_{C:D';....;D^{(r)}}^{A:B';....;B^{(r)}}\left[-z_1\left(\frac{t}{a+bt+ct^2}\right)^{\sigma_1},...,-z_r\left(\frac{t}{a+bt+ct^2}\right)^{\sigma_r}\right|$$
$$\left.\begin{matrix}[1-(a):\theta',....,\theta^{(r)}];[1-(b'):\varphi'];.....;[1-(b^{(r)}):\varphi^{(r)}]\\ [1-(c):\psi';...;\psi^{(r)}][1-(d'):\delta'];.....;[1-(d^{(r)}):\delta^{(r)}]\end{matrix}\right]dt$$

$$= \sqrt{\frac{\pi}{c}} \sum_{G,k=0}^{\infty} \sum_{g=1}^{M} \sum_{\alpha_1=0}^{[N_1/M_1]} \cdots \sum_{\alpha_s=0}^{[N_s/M_s]} \frac{(-1)^G (u_1)_k \ldots (u_p)_k}{G!\ F_g\ (v_1)_k \ldots (v_q)_k} \frac{1}{\Gamma(\gamma k+1)} \varphi(\eta_G)$$

$$.\frac{(-N_1)_{M_1\alpha_1}}{\alpha_1!} \ldots \frac{(-N_s)_{M_s\alpha_s}}{\alpha_s!} A[N_1,\alpha_1;\ldots;N_s,\alpha_s] w^k\ w_1^{\alpha_1} \ldots w_s^{\alpha_s} (b+2\sqrt{ca})^{\alpha-\sigma\eta_G-nk-\sum_{i=1}^{s} n_i\alpha_i-1}$$

$$.\frac{\Gamma(1-\alpha+\sigma\eta_G+nk+\sum_{i=1}^{s} n_i\alpha_i)}{\Gamma(\frac{3}{2}-\alpha+\sigma\eta_G+nk+\sum_{i=1}^{s} n_i\alpha_i)} F_{C+1:D';\ldots;D^{(r)}}^{A+1B';\ldots;B^{(r)}} \left[-z_1(b+2\sqrt{ca})^{-\sigma_1},\ldots,-z_r(b+2\sqrt{ca})^{-\sigma_r}\right.$$

$$\left.\begin{matrix}[1-\alpha+\sigma\eta_G+nk+\sum_{i=1}^{s} n_i\alpha_i;\sigma_i;\ldots;\sigma_r],[1-(a):\theta';\ldots;\theta^{(r)}]:[1-(b')\varphi'];\ldots;[1-(b^{(r)}):\varphi^{(r)}] \\ [1-(c);\psi'\ldots;\psi^{(r)}],[\frac{3}{2}-\alpha+\sigma\eta_G+nk+\sum_{i=1}^{s} n_i\alpha_i;\sigma_1;\ldots;\sigma_r],[1-(d'):\delta'];\ldots;[1-(d^{(r)}):\delta^{(r)}]\end{matrix}\right], \quad (3.1)$$

providing that $Re(a) > 0, Re(b) > 0, c > 0$, the series on the right side exists.
II. Taking $\theta',\ldots,\theta^{(r)} = \varphi',\ldots,\varphi^{(r)} = \psi',\ldots,\psi^{(r)} = \delta',\ldots\delta^{(r)} = \sigma_1,\ldots,\sigma_r = \alpha',\ldots,\alpha^{(r)}$ in (2.1), we get the following transformation :

$$\int_0^\infty t^{1-\alpha}(a+bt+ct^2)^{\alpha-3/2} H_{P,Q}^{M,N}\left[\left(\frac{t}{a+bt+ct^2}\right)^\sigma \middle| \begin{matrix}(e_p,E_p)\\(f_Q,F_Q)\end{matrix}\right]$$

$$._p\overset{\gamma}{M}_q\left[w\left(\frac{t}{a+bt+ct^2}\right)^n\right] S\ \begin{matrix}M_1,\ldots,M_s\\N_1,\ldots,N_s\end{matrix}\ \left[w_1\left(\frac{t}{a+bt+ct^2}\right)^{n_1},\ldots,w_s\left(\frac{t}{a+bt+ct^2}\right)^{n_s}\right]$$

$$.G\ \begin{matrix}0,\lambda:\ (u',v');\ldots\ldots;(u^{(r)},v^{(r)})\\A,C:[B',D'];\ldots;[B^{(r)},D^{(r)}]\end{matrix}\ \left[z_1^{1/\alpha'}\left(\frac{t}{a+bt+ct^2}\right),\ldots,z_r^{1/\alpha^{(r)}}\left(\frac{t}{a+bt+ct^2}\right) \middle| \begin{matrix}(a):(b');\ldots;(b^{(r)})\\(c):(d');\ldots;(d^{(r)})\end{matrix}\right] dt$$

$$= \sqrt{\frac{\pi}{c}} \sum_{G,k=0}^{\infty} \sum_{g=1}^{M} \sum_{\alpha_1=0}^{[N_1/M_1]} \cdots\cdot \sum_{\alpha_s=0}^{[N_s/M_s]} \frac{(-1)^G (u_1)_k \ldots (u_p)_k}{G!\ F_g\ (v_1)_k \ldots (v_q)_k} \frac{1}{\Gamma(\gamma k+1)} \varphi(\eta_G)$$

$$\frac{(-N_1)_{M_1\alpha_1}}{\alpha_1!} \ldots \frac{(-N_s)_{M_s\alpha_s}}{\alpha_s!} A[N_1,\alpha_1;\ldots;N_s,\alpha_s] w^k w_1^{\alpha_1} \ldots w_s^{\alpha_s}$$

$$(b+2\sqrt{ca})^{\alpha-\sigma\eta_G-nk-\sum_{i=1}^{s} n_i\alpha_i-1}$$

$$.G\ \begin{matrix}0,\lambda+1:\ (u',v');\ldots;(u^{(r)},v^{(r)})\\A+1,C+1:[B',D'];\ldots;[B^{(r)},D^{(r)}]\end{matrix}\ \left[z_1^{1/\alpha'}(b+2\sqrt{ca})^{-1},\ldots,z^{1/\alpha^{(r)}}(b+2\sqrt{ca})^{-1}\right.$$

$$\left|\begin{matrix}[\alpha-\sigma\eta_G-nk-\sum_{i=1}^{s} n_i\alpha_i],(a):(b');\ldots;(b^{(r)})]\\ [(c),[\alpha-\sigma\eta_G-nk-\sum_{i=1}^{s} n_i\alpha_i-\frac{1}{2}]:(d');\ldots;(d^{(r)})\end{matrix}\right], \quad (3.2)$$

$\mathrm{Re}(a) > 0, \mathrm{Re}(b) > 0, c > 0; \alpha^{(i)} > 0 (i = 1,\ldots,r), 2(u^{(i)} + v^{(i)}) < (A + C + B^{(i)} + D^{(i)})$
$|\arg(z_i)| < \left[u^{(i)} + v^{(i)} - \frac{A}{2} - \frac{C}{2} - \frac{B^{(i)}}{2} - \frac{D^{(i)}}{2}\right]\pi, p \leqslant q, |w| \leqslant 1$ *and*
$\sigma\left\{\min_{1\leqslant j\leqslant M}[\mathrm{Re}(f_j/F_j)]\right\} + \sum_{i=1}^{r}\left\{\min_{1\leqslant j\leqslant u^{(i)}}[\mathrm{Re}(d_j^{(i)})]\right\} > \alpha - 2.$

III. When $\lambda = A = C = 0$ in (2.1), we have the following

$$\int_0^\infty t^{1-\alpha}(a+bt+ct^2)^{\alpha-3/2} H_{P,Q}^{M,N}\left[\left(\frac{t}{a+bt+ct^2}\right)^\sigma \middle| \begin{matrix}(e_p,E_p)\\(f_Q,F_Q)\end{matrix}\right]$$

$$_p\overset{\gamma}{M}_q\left[w\left(\frac{t}{a+bt+ct^2}\right)^n\right] S\ \begin{matrix}M_1,\ldots,M_s\\N_1,\ldots,N_s\end{matrix}\ \left[w_1\left(\frac{t}{a+bt+ct^2}\right)^{n_1},\ldots,w_s\left(\frac{t}{a+bt+ct^2}\right)^{n_s}\right]$$

$$.\prod_{i=1}^{r} H_{B^{(i)},D^{(i)}}^{u^{(i)},v^{(i)}}\left[z_i\left(\frac{t}{a+bt+ct^2}\right)^{\sigma_i} \middle| \begin{matrix}[(b^{(i)}):\varphi^{(i)}]\\ [(d^{(i)}):\delta^{(i)}]\end{matrix}\right] dt$$

$$= \sqrt{\tfrac{\pi}{c}} \sum_{G,k=0}^{\infty} \sum_{g=1}^{M} \sum_{\alpha_1=0}^{[N_1/M_1]} \cdots \sum_{\alpha_s=0}^{[N_s/M_s]} \frac{(-1)^G (u_1)_k \ldots (u_p)_k}{G!\ F_g\ (v_1)_k \ldots (v_q)_k} \frac{1}{\Gamma(\gamma k+1)} \varphi(\eta_G)$$
$$\frac{(-N_1)_{M_1\alpha_1}}{\alpha_1!} \ldots \frac{(-N_s)_{M_s\alpha_s}}{\alpha_s!} A[N_1,\alpha_1;\ldots;N_s,\alpha_s] w^k w_1^{\alpha_1} \ldots w_s^{\alpha_s}$$
$$(b+2\sqrt{ca})^{\alpha-\sigma\eta_G-nk-\sum_{i=1}^{s} n_i\alpha_i-1}$$
$$.H \begin{matrix} 0,1:\ (u',v');\ldots;(u^{(r)},v^{(r)}) \\ 1,1:[B',D'];\ldots;[B^{(r)},D^{(r)}] \end{matrix} \left[\begin{matrix} z_1(b+2\sqrt{ca})^{-\sigma_1} \\ \vdots \\ z_r(b+2\sqrt{ca})^{-\sigma_r} \end{matrix} \middle| \begin{matrix} [\alpha-\sigma\eta_G-nk-\sum_{i=1}^{s} n_i\alpha_i:\sigma_1;\ldots;\sigma_r]:[(b')\varphi'];\ldots;[(b^{(r)}):\varphi^{(r)}] \\ [\alpha-\sigma\eta_G-nk-\sum_{i=1}^{s} n_i\alpha_i-\frac{1}{2}:\sigma_1;\ldots;\sigma_r]:[(d')\delta'];\ldots;[(d^{(r)}):\delta^{(r)}] \end{matrix} \right] \quad (3.3)$$

valid under the same conditions as obtained from (2.1).

IV. When $\gamma = 1$ in (2.1), we have the following transformation :

$$\int_0^{\infty} t^{1-\alpha}(a+bt+ct^2)^{\alpha-3/2} H_{P,Q}^{M,N}\left[\left(\tfrac{t}{a+bt+ct^2}\right)^{\sigma} \middle| \begin{matrix}(e_p,E_p)\\(f_Q,F_Q)\end{matrix}\right]$$
$${}_pF_q\left[w\left(\tfrac{t}{a+bt+ct^2}\right)^{n}\right] S \begin{matrix} M_1,\ldots,M_s \\ N_1,\ldots,N_s \end{matrix} \left[w_1\left(\tfrac{t}{a+bt+ct^2}\right)^{n_1},\ldots,w_s\left(\tfrac{t}{a+bt+ct^2}\right)^{n_s}\right]$$
$$.H\left[z_1\left(\tfrac{t}{a+bt+ct^2}\right)^{\sigma_1},\ldots,z_r\left(\tfrac{t}{a+bt+ct^2}\right)^{\sigma_r}\right] dt$$
$$= \sqrt{\tfrac{\pi}{c}} \sum_{G,k=0}^{\infty} \sum_{g=1}^{M} \sum_{\alpha_1=0}^{[N_1/M_1]} \cdots \sum_{\alpha_s=0}^{[N_s/M_s]} \frac{(-1)^G (u_1)_k \ldots (u_p)_k}{G!\ F_g\ (v_1)_k \ldots (v_q)_k} \frac{1}{k!}$$
$$\frac{(-N_1)_{M_1\alpha_1}}{\alpha_1!} \ldots \frac{(-N_s)_{M_s\alpha_s}}{\alpha_s!} A[N_1,\alpha_1;\ldots;N_s,\alpha_s] w^k w_1^{\alpha_1} \ldots w_s^{\alpha_s}$$
$$(b+2\sqrt{ca})^{\alpha-\sigma\eta_G-nk-\sum_{i=1}^{s} n_i\alpha_i-1}$$
$$.H \begin{matrix} 0,\lambda+1:\ (u',v');\ldots;(u^{(r)},v^{(r)}) \\ A+1,C+1:[B^{'},D'];\ldots;[B^{(r)},D^{(r)}] \end{matrix} \left[\begin{matrix} z_1(b+2\sqrt{ca})^{-\sigma_1} \\ \vdots \\ z_r(b+2\sqrt{ca})^{-\sigma_r} \end{matrix} \middle| \begin{matrix} [\alpha-\sigma\eta_G-nk-\sum_{i=1}^{s} n_i\alpha_i:\sigma_1;\ldots;\sigma_r], \\ [(c):\psi',\ldots,\psi^{(r)}], \end{matrix} \right.$$
$$\left. \begin{matrix} [(a):\theta',\ldots,\theta^{(r)}]:[(b')\varphi'];\ldots;[(b^{(r)}):\varphi^{(r)}] \\ [\alpha-\sigma\eta_G-nk-\sum_{i=1}^{s} n_i\alpha_i-\frac{1}{2}:\sigma_1;\ldots;\sigma_r]:[(d')\delta'];\ldots;[(d^{(r)}):\delta^{(r)}] \end{matrix} \right] \quad (3.4)$$

valid under the same conditions as needed for (2.1).

V. Replacing $n = 0$and $N_1 \ldots N_s$ by N in the result (2.1) reduces to the known result given in [2], after a little simplification.

VI. Taking $n = 0, N_i \to 0, (i = 1, \ldots .s), a = 0, c = 1$, then result in (2.1) reduces to the known result after a little simplification obtained by Goyal and Mathur [5].

VII. If $n = 0$ and $M_i, N_i \to 0\,(i = 1, 2, 3\ldots)$ the result in (2.1) reduces to the known result with a slight modification derived by Gupta and Jain [6].

VIII. When $n = 0$, the results in (2.1), (3.1), (3.2) and (3.3) reduce to the known results with a slight simplification recently obtained by Chaurasia and Shekhawat [3].

References

[1] Braaksma, B.L.J.- Asymptotic expansions and analytic continuation for a class of Barnes integrals, Compositio Math., 15 (1963), 236-341.

[2] Chaurasia, V.B.L. and Godika, A.- An integral involving certain special functions, Bull. Cal. Math. Soc., 91 (1999), 337-342.
[3] Chaurasia, V.B.L. and Shekhawat, A.S.- An integral involving general polynomials and the H-function of several complex variables, Tamkang J. Math., 36 (2005), 255-260.
[4] Fox, C.- The G and H-function as symmetrical Fourier kernels, Trans. Amer. Math. Soc., 98 (1961), 395-429.
[5] Goyal, S.P. and Mathur, S.L.- On integrals involving the H-function of two variables, Indian J. Pure Appl. Math., 7 (1976), 347-358.
[6] Gupta, K.C. and Jain, R.- An integral involving a general polynomials and product of Fox's H-function having general arguments, Ganita Sandesh, 3 (1989), 64-67.
[7] Sharma, M.- Fractional integration and fractional differentiation of the M-series, Frac. Cal. Appl. Anal., 11 (2008), 187-191
[8] Saxena, R.K.- An integral involving G-function, Proc. Nat. Inst. Sci. India, 26A (1960), 661-664.
[9] Skibinski, P.- Some expansion theorem for the H-functions, Ann. Polon. Math., 23 (1976), 1-6.
[10] Srivastava, H.M.- A multilinear generating function for the Kohnauser sets of bi-orthogonal polynomials suggested by the Laguerre polynomials, Pacific J. Math., 117 (1985), 183-191.
[11] Srivastava, H.M. and Panda, R.- Some bilateral generating function for a class of generalized hypergeometric polynomials, J. Reine. Angew. Math., 283/284 (1996), 265-274.

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