SCIENTIA
Series A: *Mathematical Sciences*, Vol. 17 (2009), 1–8
Universidad Técnica Federico Santa María
Valparaíso, Chile
ISSN 0716-8446

Spaces with compact-countable weak-bases

Zhaowen Li[1,a] and Qingguo Li[1]

Abstract. In this paper, we establish the relationships between spaces with a compact-countable weak-base and spaces with various compact-countable networks, and give two mapping theorems on spaces with compact-countable weak-bases.

Weak-bases and g-first countable spaces were introduced by A.V.Arhangel'skii [1]. Spaces with a point-countable weak-base were discussed in [5,6], and spaces with a locally countable weak-base were discussed in [7,8,9]. In this paper, we shall investigate spaces with a compact-countable weak-base, establish the relationships between spaces with a compact-countable weak-base and spaces with various compact-countable networks, and give two mapping theorems on spaces with compact-countable weak-bases.

We assume that spaces are regular and T_1, and mapping are continuous and onto.

Definition 1. Let $\mathcal{P}$ be a family of subsets of a space X, put

$$\mathcal{P}^{<\omega} = \{\mathcal{P}' \subset \mathcal{P} : |\mathcal{P}'| < \omega\}.$$

(1) $\mathcal{P}$ is compact-countable in X if for each compact subset K of X, only countably many members of $\mathcal{P}$ intersect K.

(2) $\mathcal{P}$ is a k-network[11] for X if for each compact subset K of X and its open neighborhood V, there exists $\mathcal{P}' \in \mathcal{P}^{<\omega}$ such that $K \subset \cup\mathcal{P}' \subset V$.

(3) $\mathcal{P}$ is a cs-network[12] for X if for each $x \in X$, its open neighborhood V and a sequence $\{x_n\}$ converging to x, there exists $P \in \mathcal{P}$ such that $\{x_n : n \geqslant m\} \cup \{x\} \subset P \subset V$ for some $m \in N$.

Definition 2.[13] For a space X and $x \in P \subset X$, P is a sequential neighborhood of x in X if, whenever $\{x_n\}$ is a sequence converging to x in X, then $x_n \in P$ for all but finitely many $n \in N$. P is a sequential open set of X if for each $x \in P$, P is a sequential neighborhood of x in X.

A space X is a sequential space if each sequential open set of X is open in X.

2000 *Mathematics Subject Classification.* 54D50; 54D99; 54E40.

Key words and phrases. Weak-bases; sn-networks; cs-networks; k-networks; k-spaces; α_4-spaces; g-first countable spaces.

The project is supported by the NSF of China (No. 10471035, 10671211), the NSF of Hunan Province in China (No. 06JJ20046) and the NSF of Education Department of Hunan Province in China (No. 06C461).

Definition 3. Let $\mathcal{P} = \cup\{\mathcal{P}_x : x \in X\}$ be a family of subsets of a space X satisfying that for each $x \in X$,

(1) $\mathcal{P}_x$ is a network of x in X,

(2) If $U, V \in \mathcal{P}_x$, then $W \subset U \cap V$ for some $W \in \mathcal{P}_x$.

$\mathcal{P}$ is a weak-base for X[1] if $G \subset X$ is open in X if and only if for each $x \in G$, there exists $P \in \mathcal{P}_x$ such that $P \subset G$. $\mathcal{P}$ is an sn-network[5] (i.e., an sequential neighborhood network) for X if each element of $\mathcal{P}_x$ is a sequential neighborhood of x in X, here $\mathcal{P}_x$ is an sn-network of x in X.

A space X is a g-first countable space[1] (resp. an sn-first countable space[10]) if X has a weak-base (resp. an sn-network) $\mathcal{P}$ such that each $\mathcal{P}_x$ is countable.

For a space, weak-base $\Rightarrow$ sn-network $\Rightarrow$ cs-network. An sn-network for a sequential space is a weak-base [5].

Definition 4. Call a subspace of a space a fan (at a point x) if it consists of a point x, and a countably infinite family of disjoint sequences converging to x. Call a subset of a fan a diagonal if it is a convergent sequence meeting infinitely many of the sequences converging to x and converges to some point in the fan.

(1) A space X is an α_1-space[2,3] if $T = \{x\} \cup (\cup\{T_n : n \in N\})$ is a fan at x of X, where each sequence T_n converges to x, then there exists a sequence S converging to x such that $T_n \setminus S$ is finite for each $n \in N$.

(2) A space X is an α_4-space[2,3] if every fan at x of X has a diagonal converging to x.

It is clear that [10] k-space $\Leftarrow$ sequential space $\Leftarrow$ g-first countable space $\Rightarrow$ sn-first countable space $\Rightarrow$ α_1-space $\Rightarrow$ α_4-space.

Lemma 5. The following are equivalent for a space X:

(1) X has a compact-countable sn-network.

(2) X is an sn-first countable space with a compact-countable cs-network.

(3) X is a α_1-space with a compact-countable cs-network.

(4) X is a α_4-space with a compact-countable cs-network.

Proof. (1)$\Rightarrow$(2)$\Rightarrow$(3)$\Rightarrow$(4) are clear. We show that (4)$\Rightarrow$(1). Suppose X is a α_4-space with a compact-countable cs-network $\mathcal{P}$. Let $\mathcal{P}_1 = \{\cap\mathcal{P}' : \mathcal{P}' \in \mathcal{P}^{<\omega}\}$. Since $\mathcal{P} \subset \mathcal{P}$, then $\mathcal{P}_1$ is a cs-network for X. For each compact $K \subset X$, since $\mathcal{P}$ is compact-countable in X, then there exists $\{P_n : n \in N\} \subset \mathcal{P}$ such that $K \cap P = \emptyset$ for each $P \in \mathcal{P} \setminus \{P_n : n \in N\}$. For each $\mathcal{P}' \in \mathcal{P}^{<\omega}$, if $\mathcal{P}' \cap (\mathcal{P} \setminus \{P_n : n \in N\}) \neq \emptyset$, then $K \cap (\cap\mathcal{P}') = \emptyset$; if $\mathcal{P}' \cap (\mathcal{P} \setminus \{P_n : n \in N\}) = \emptyset$, then $\mathcal{P}' \subset \{P_n : n \in N\}$, so $\mathcal{P}' \in \{P_n : n \in N\}^{<\omega}$. Because $|\{P_n : n \in N\}^{<\omega}| < \omega$ and $\{\mathcal{P}' \in \mathcal{P}^{<\omega} : K \cap (\cap\mathcal{P}') \neq \emptyset\} \subset \{P_n : n \in N\}^{<\omega}$, then $\mathcal{P}_1$ is compact-countable in X. Hence $\mathcal{P}_1$ is a compact-countable cs-network for X which is closed under finite intersections. So we may assume that X has a compact-countable cs-network $\mathcal{P}$ which is closed under finite intersections. By Theorem 3.13 in [7], X is sn-first countable. For each $x \in X$, let $\{B(n, x) : n \in N\}$ be a decreasing sn-network of x in X. Put

$$\mathcal{F}_x = \{P \in \mathcal{P} : B(n, x) \subset P \text{ for some } n \in N\}.$$
$$\mathcal{F} = \cup\{\mathcal{F}_x : x \in X\}$$

Obviously, $x \in \cap \mathcal{F}_x$ and $\mathcal{F}_x$ is closed under finite intersections. Then $\mathcal{F}$ satisfies Definition 3 (1),(2). We claim that each element of $\mathcal{F}_x$ is a sequential neighborhood at x in X. Otherwise, there exists $P \in \mathcal{F}_x$ such that P is not a sequential neighborhood of x in X. Then there exists a sequence $\{x_n\}$ converging to x such that for each $k \in N$, $\{x_n : n > k\} \not\subset P$. Take $x_{n_1} \in \{x_n : n > 1\} \setminus P$, then there exists a subsequence $\{x_{n_k}\}$ of $\{x_n\}$ such that each $x_{n_{k+1}} \in \{x_n : n > n_k\} \setminus P$. Obviously, x_{n_k} converges to x. Since $P \in \mathcal{F}_x$, then $B(m,x) \subset P$ for some $m \in N$. Because $B(m,x)$ is a sequential neighborhood of x in X, then $\{x\} \cup \{x_{n_k} : k \geqslant j\} \subset B(m,x)$ for some $j \in N$, and so $\{x_{n_k} : k \geqslant j\} \subset P$, a contradiction. Hence $\mathcal{F}$ is an sn-network for X. Obviously, $\mathcal{F} \subset \mathcal{P}$. Therefore $\mathcal{F}$ is a compact-countable sn-network for X.

Lemma 6. Every compact-countable cs-network for a space X is a k-network for X.

Proof. Let $\mathcal{P}$ be a compact-countable cs-network for X. We will show that $\mathcal{P}$ is a k-network for X. Suppose $K \subset V$ with K non-empty compact and V open in X. Put

$$\mathcal{A} = \{P \in \mathcal{P} : P \cap K \neq \emptyset \text{ and } P \subset V\},$$

then $\mathcal{A} = \cup\{\mathcal{A}_n : n \in N\}$ is countable. Denote $\mathcal{A} = \{P_i : i \in N\}$, then $K \subset \bigcup\limits_{i \leqslant n} P_i$ for some $n \in N$. Otherwise, $K \not\subset \bigcup\limits_{i \leqslant n} P_i$ for each $n \in N$, so choose $x_n \in K \setminus \bigcup\limits_{i \leqslant n} P_i$. Because $\{P \cap K : P \in \mathcal{P}\}$ is a countable cs-network for a subspace K and a compact space with a countable network is metrizable, then K is a compact metrizable space. Thus $\{x_n\}$ has a convergent subsequence $\{x_{n_k}\}$, where $x_{n_k} \to x$. Obviously, $x \in K$, so V is an open neighborhood of x in X. Since $\mathcal{P}$ is a cs-network for X, then there exist $m \in N$ and $P \in \mathcal{P}$ such that $\{x_{n_k} : k \geqslant m\} \cup \{x\} \subset P \subset V$. Now, $P = P_j$ for some $j \in N$. Take $l \geqslant m$ such that $n_l \geqslant j$, then $x_{n_l} \in P_j$. This is a contradiction.

Remark 7. By Lemma 6, X has a compact-countable cs-network $\Rightarrow$ X has a compact-countable k-network. But X has a point-countable cs-network $\not\Rightarrow$ X has a compact-countable k-network because X has a point-countable sn-network $\not\Rightarrow$ X has a point-countable k-network, for example, the Stone-Čech compactification βN.

On the other hand, let X be S_{ω_1}, by Proposition 2.7.21 in [14], X is a Lašnev space and has no point-countable cs^*-networks. Then X has a σ-hereditarily closure-preserving k-network, so X has a σ-compact-finite k-network (see [19, Proposition 2]). This implies that X has a compact-countable k-network. Hence X has a compact-countable k-network $\not\Rightarrow$ X has a compact-countable cs-network.

Theorem 8. The following are equivalent for a space X:
(1) X has a compact-countable weak-base.
(2) X is a k-space with a compact-countable sn-network.
(3) X is a k-and sn-first countable space with a compact countable cs-network.
(4) X is a k-and α_1-space with a compact-countable cs-network.
(5) X is a k-and α_4-space with a compact-countable cs-network.
Proof. (1)$\Rightarrow$(2) is obvious.

(2)⇒(1). Suppose X is a k-space with a compact countable sn-network $\mathcal{P}$, then $\mathcal{P}$ is a compact countable cs-network for X. By Lemma 6, X has a compact countable k-network. Since a k-space with a point countable k-network is sequential ([15, Corollary 3.4]), then X is a sequential space. Thus $\mathcal{P}$ is a weak-base for X. Hence X has a compact-countable weak-base.

(2) ⇔ (3) ⇔ (4) ⇔ (5) hold by Lemma 5.

By Lemma 6 and Theorem 8, we have

Theorem 9 For a space X, (1) ⇔ (2) ⇒ (3) hold.

(1) X has a compact-countable weak-base.

(2) X is g-first countable space with a compact-countable cs-network.

(3) X is g-first countable space with a compact-countable k-network.

In the following, we recall some definitions,

For a subset family $\mathcal{F}$ of a space X and $A \subset X$, $\mathcal{F}$ is a minimal cover of A if $A \subset \cup\mathcal{F}$ and $A \not\subset \cup\mathcal{F}'$ for each proper subset $\mathcal{F}'$ of $\mathcal{F}$.

Let $f : X \to Y$ be a mapping. f is a k-mapping if for each compact subset K of Y, $f^{-1}(K)$ is compact in X. f is a cs-mapping[17] if for each compact subset C of Y, $f^{-1}(C)$ is separable in X. f is a 1-sequence-covering mapping[5] if for each $y \in Y$, there exists $x \in f^{-1}(y)$ satisfying the following condition: whenever $\{y_n\}$ is a sequence of Y converging to a point y in Y, there exists a sequence $\{x_n\}$ of X converging to a point x in X such that each $x_n \in f^{-1}(y_n)$.

Obviously, perfect mappings ⇒ k-mappings.

Lemma 10. Spaces with a compact-countable k-network are preserved under k-mappings.

Proof. Let $f : X \to Y$ be a k-mapping such that X has a compact-countable k-network $\mathcal{P}$. For each $\mathcal{F} \in \mathcal{P}^{<\omega}$, put

$$M(\mathcal{F}) = \{y \in Y : \mathcal{F} \text{ is a mininal cover of } f^{-1}(y)\}.$$

Let $\mathcal{R} = \{M(\mathcal{F}) : \mathcal{F} \in \mathcal{P}^{<\omega}\}$. For each compact $K \subset Y$, since f is a k-mapping, then $f^{-1}(K)$ is compact in X. Since $\mathcal{P}$ is compact-countable in X, then there exists $\{P_n : n \in N\} \subset \mathcal{P}$ such that $f^{-1}(K) \cap P = \emptyset$ for each $P \in \mathcal{P} \setminus \{P_n : n \in N\}$. Denote $\mathcal{P}_1 = \{P_n : n \in N\}$. For each $P \in \mathcal{P} \setminus \mathcal{P}_1$ and each $\mathcal{F} \in \mathcal{P}^{<\omega}$ with $P \in \mathcal{F}$, we claim that $K \cap M(\mathcal{F}) = \emptyset$. Otherwise. There exist $P \in \mathcal{P} \setminus \mathcal{P}_1$ and $\mathcal{F} \in \mathcal{P}^{<\omega}$ such that $P \in \mathcal{F}$ and $K \cap M(\mathcal{F}) \neq \emptyset$. Take $y \in K \cap M(\mathcal{F})$. Then $f^{-1}(y) \subset f^{-1}(K)$. Since $f^{-1}(K) \cap P = \emptyset$, thus $f^{-1}(y) \cap P = \emptyset$. Because $y \in M(\mathcal{F})$, then $f^{-1}(y) \subset \cup\mathcal{F}$. Hence $f^{-1}(y) \subset \cup(\mathcal{F} \setminus \{P\})$, a contradiction. Because $|\mathcal{P}_1^{<\omega}| < \omega$ and $\{\mathcal{F} \in \mathcal{P}^{<\omega} : K \cap M(\mathcal{F}) \neq \emptyset\} \subset \mathcal{P}_1^{<\omega}$, thus $\mathcal{R}$ is compact-countable in Y. So it suffices to show that $\mathcal{R}$ is a k-network for Y. For $K \subset V$ with K compact and V open in Y, $f^{-1}(K) \subset f^{-1}(V)$ with $f^{-1}(K)$ compact and $f^{-1}(V)$ open in X, since $\mathcal{P}$ is a k-network for X, then $f^{-1}(K) \subset \cup\mathcal{P}' \subset f^{-1}(V)$ for some $\mathcal{P}' \in \mathcal{P}^{<\omega}$. Put

$$\mathcal{R}' = \{M(\mathcal{F}) : \mathcal{F} \subset \mathcal{P}'\}.$$

For each $y \in K$, $f^{-1}(y) \subset \cup\mathcal{P}'$. Suppose $\mathcal{F}_1 \subset \mathcal{P}'$ is a minimal cover of $f^{-1}(y)$, then $y \in M(\mathcal{F}_1)$, and so $y \in \cup\mathcal{R}'$. Hence $K \subset \cup\mathcal{R}'$. For each $\mathcal{F} \subset \mathcal{P}'$ and each $y \in M(\mathcal{F})$,

$f^{-1}(y) \subset \cup\mathcal{F} \subset \cup\mathcal{P}' \subset f^{-1}(V)$, then $y \in V$. So $M(\mathcal{F}) \subset V$. Hence $\cup\mathcal{R}' \subset V$. This shows that $\mathcal{R}$ is a compact-countable k-network for Y.

Corollary 11. Spaces with a compact-countable k-network are preserved under perfect mappings.

By Lemma 6 and Lemma 10, we have following mapping theorem on spaces with a compact-countable weak-base.

Theorem 12. Let $f : X \to Y$ be k-mapping such that X has a compact-countable weak-base. Then Y has a compact-countable k-network.

Remark 13. The space of Example 2.14(1) in [16] has a countable weak-base, but its image under a perfect mapping is not g-first countable. Thus spaces with a compact-countable weak-base are not necessarily preserved under perfect mappings.

Further, from Alexandroff's sorting idea of spaces by means of mappings, we give second mapping theorem on spaces with a compact-countable weak-base, and establish relationships between metric spaces and spaces with compact-countable weak-bases.

Proposition 14. A space X has a compact-countable sn–network if and only if X is a 1-sequence-covering cs-image of a metric space.

Proof. Necessity. Suppose $\mathcal{P}$ is a sn–network for X. Denote $\mathcal{P} = \{P_\alpha : \alpha \in A\}$. For each $i \in N$, let A_i be a copy of A, and it is endowed with discrete topology. Put

$$M = \{\beta = (\alpha_i) \in \prod_{i\in N} A_i : \{P_{\alpha_i} : i \in N\} \text{ is a network of some point } x(\beta) \text{ in } X\},$$

and give M the subspace topology induced from the product topology of the product space $\prod_{i\in N} A_i$. The point $x(\beta)$ is unique in X because X is Hausdroff. We define $f : M \to X$ by $f(\beta) = x(\beta)$. Obviously, M is a metric space. By the proof of Theorem 1, we can prove that f is a cs-mapping. For each $x \in X$, let $\{P_{\alpha_i} : i \in N\} \subset \mathcal{P}$ be a sequential neighborhood network of x in X. Denote $\beta = (\alpha_i)$, then $\beta \in f^{-1}(x)$. For each $n \in N$, put $R_n = \{(\gamma_i) \in M : \gamma_i = \alpha_i \text{ for each } i \leqslant n\}$. Then $\{R_n : n \in N\}$ is a decrease neighborhood base of β in M and $f(R_n) = \bigcap_{i\leqslant n} P_{\alpha_i}$ for each $n \in N$. In fact, assume $\gamma = (\gamma_i) \in R_n$, then $f(\gamma) \in \bigcap_{i\in N} P_{\gamma_i} \subset \bigcap_{i\leqslant n} P_{\alpha_i}$. Hence $f(R_n) \subset \bigcap_{i\leqslant n} P_{\alpha_i}$. And assume $z \in \bigcap_{i\leqslant n} P_{\alpha_i}$, then there exists $\{P_{\delta_i} : i \in N\} \subset \mathcal{P}$ such that $\delta_i = \alpha_i$ when $i \leqslant n$ and $\{P_{\delta_i} : i \in N\}$ is a network of z in X. Put $\delta = (\delta_i)$, then $\delta \in R_n$ and $f(\delta) = z \in f(R_n)$, and hence $\bigcap_{i\leqslant n} P_{\alpha_i} \subset f(R_n)$. Therefore, $f(R_n) = \bigcap_{i\leqslant n} P_{\alpha_i}$. Now, assume $x_j \to x$ in X. For each $n \in N$, since $f(R_n)$ is a sequential neighborhood of x, there exists $i(n) \in N$ such that $x_i \in f(R_n)$ when $i \geqslant i(n)$. Hence $f^{-1}(x_i) \cap R_n \neq \emptyset$. We can assume $1 < i(n) < i(n+1)$. For each $j \in N$, take $\beta_j \in f^{-1}(x_j)$ when $j < i(n)$ and take $\beta_j \in f^{-1}(x_j) \cap R_n$ when $i(n) \leqslant j < i(n+1)$, then $\beta_j \to \beta$ in M. Therefore, f is 1-sequence-covering mapping.

Sufficiency. Suppose $f : M \to X$ is a 1-sequence-covering cs-mapping, where M is a metric space. Let $\mathcal{B}$ be a σ-locally finite base for M. For each $x \in X$, there exists

$\beta_x \in f^{-1}(x)$ satisfying definition1 (5). Put

$$\mathcal{P}_x = \{f(B) : \beta_x \in B \in \mathcal{B}\},$$
$$\mathcal{P} = \cup\{\mathcal{P}_x : x \in X\},$$

it is easy to prove that $\mathcal{P}$ is a compact-countable sn–network for X.

Theorem 15. X has a compact-countable weak-base if and only if X is a 1-sequence-covering and quotient cs-image of a metric space.

Proof. Necessity. Suppose X has a compact-countable weak-base, then X is a sequential space with a compact-countable sequential neighborhood network by proposition 1.6.15 and corollary 1.6.18 of [2]. Hence X is a 1-sequence-covering cs-image of a metric space by Theorem 3. Thus this 1-sequence-covering mapping is a quotient mapping by Lemma 2.1 of [5].

Sufficiency. Suppose X is a 1-sequence-covering and quotient cs-image of a metric space, then X is a sequential space with a compact-countable sequential neighborhood network $\mathcal{P}$. It is easy to prove that $\mathcal{P}$ is a compact-countable weak-base for X.

Finally, we give two examples.

Example 16 A separable, regular space X has a point-countable weak-base but no a compact-countable weak-base.

Let

$$S = \left\{\frac{1}{n} : n \in N\right\} \cup \{0\}, \quad X = [0,1] \times S,$$

and let

$$Y = [0,1] \times \{\frac{1}{n} : n \in N\}$$

have the usual Euclidean topology as a subspace of $[0,1] \times S$. Define a typical neighborhood of $(t,0)$ in X to be of the form

$$\{(t,0)\} \cup \left(\bigcup_{k \geqslant n} V(t,1/k)\right), \quad n \in N,$$

where $V(t,1/k)$ is a neighborhood of $(t,1/k)$ in $[0,1] \times \{1/k\}$. Put

$$M = (\oplus_{n\in N}[0,1] \times \{1/n\}) \oplus (\oplus_{t\in[0,1]}\{t\} \times S),$$

and define f from M onto X such that f is an obvious mapping.

Then f is a compact-covering, quotient, two-to-one mapping from the compact compact metric space M onto separable, regular, non-Lindelöf, k-space X (see Example 2.8.16 of [14] or Example 9.3 of [15]). It is easy to check that f is a 1-sequence-covering mapping. By Theorem 2.5 in [5], X has a point-countable weak-base.

X has no compact-countable k-network. In fact. Suppose $\mathcal{P}$ is a compact-countable k-network for X. Put

$$\mathcal{F} = \{\{(t,0)\} : t \in [0,1]\} \cup \{P \cap Y : P \in \mathcal{P}\}.$$

Since $[0,1] \times \{0\}$ is a closed discrete subspace of X, then $\mathcal{F}$ is a k-network for X. But Y is a σ-compact subspace of X. Thus $\{P \cap Y : P \in \mathcal{P}\}$ is countable, and so $\mathcal{F}$ is star-countable. Since a regular, k-space with a star-countable k-network is a $\aleph_0$-space(see

[18]), then X is a Lindelöf space, a contradiction. Thus X has no compact-countable k-network. By Theorem 9, X has no compact-countable weak-base.

Example 17 A paracompact space X has a compact-countable weak-base but no locally countable weak-base.

Let X be a paracompact space with a point-countable base, and not metrizable. Then X has a compact-countable base, and so X has a compact-countable weak-base. But X is not a 1-sequence-covering ss-image of a metric space because X is not a metric space. Thus X has no a locally countable weak-base by Theorem 2.1 in [9].

Example 18 A g-first countable space X with a σ-compact finite k-network $\not\Longrightarrow$ X has a point-countable weak base.

Let the space X be example 9.8 in [15], it is easy to see that X is g-first countable and has a σ-compact-finite k-network. So X has a compact-countable k-network. But X does not have a point-countable weak base(see [6]).

This example illustrates: (3) $\not\Longrightarrow$ (1) in Theorem 9.

References

[1] A.V.Arhangel'skii, Mappings and spaces, Russian Math. Surveys, 21(1966), 115-162.
[2] A.V.Arhangel'skii, The frequency spectrum of a topological space and the classification of spaces, Soviet Math. Dokl., 13(1972), 265-268.
[3] A.V.Arhangel'skii, The frequency spectrum of a topological space and the product operation, Trans. Moscow Math. Soc., 2(1981), 163-200.
[4] F.Siwiec, On defining a space by a weak-base, Pacific J. Math., 52(1974), 233-245.
[5] S.Lin, On sequence-covering s-mappings, Adv. in Math., 25(1996), 548-551.
[6] S.Lin, Y.Tanaka, Point-countable k-networks, close maps, and related results, Topology Appl., 59(1994), 79-86.
[7] S.Lin, Z.Li, J.Li, C.Liu, On ss-mappings, Northeastern Math. J., 9(1993), 521-524.
[8] C.Liu, M.Dai, Spaces with a locally countable weak-base. Math. Japonica, 41(1995), 261-267.
[9] J.Li, S.Jiang, On spaces with a locally countable weak-base, Far East J. Math. Sci., Special Volume (2001), 15-24.
[10] S.Lin, A note on the Arens' space and sequential fan, Topology Appl., 8(1997), 185-196.
[11] P.O'Meara, On paracompactness in function space with open topology, Proc. Amer. Math. Soc., 29(1971), 183-189.
[12] A.Guthrie., A Characterization of $\aleph_0$-spaces, Gen. Top. Appl., 1(1971), 105-110.
[13] S.P.Franklin, Spaces in which sequences suffice, Fund. Math., 57(1965), 107-115.
[14] S.Lin, Generalized metric spaces and mappings, Chinese Scientific Publ., Beijing, 1995.
[15] G.Gruenhage, E.Michael, Y.Tanaka, Spaces determined by point-countable covers, Pacific J.Math., 113(1984), 303-332.
[16] Y.Tanaka, Symmetric spaces, g-developable space and g-metrizable space, Math. Japonica, 36(1991), 71-84.
[17] Z.Qu, Z.Gao, Spaces with compact-countable k-networks, Math. Japonica, 49(1999), 199-205.
[18] M. Sakai, On spaces with a star-countable k-network, Houston J. Math., 23(1997), 45-56.
[19] C.Liu, spaces with a σ-compact-finite k-network, Q & A in Gen. Top., 10(1992), 81-87.
[20] Z.Li, A mapping theorem on $\aleph$-spaces, Mat. Vesnik, 57(2005), 35-40.
[21] Z.Li, Spaces with σ-locally countable weak-bases, Archivum Math., 42(2006), 135-140.

Received 03 03 2008, revised 10 11 2008

[1] *Current address: College of Mathematics and Econometrics,*
Hunan University,
Changsha, Hunan 410082,
P.R.China
E-mail address: lizhaowen8846@163.com

[a] *Current address: Department of Information,*
Hunan Business College,
Changsha, Hunan 410205,
P.R.China
E-mail address: lizhaowen8846@163.com